Throughput Analysis for Coded Multichannel ALOHA Random Access

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***Abstract*— In this letter, we study coded random access with multichannel ALOHA where incremental redundancy-based re-transmissions are employed. A closed-form expression for the throughput is derived, which allows optimizing key parameters for given conditions. A scaling property of multichannel ALOHA is also shown with an asymptotic throughput, which demonstrates that the throughput grows linearly with the number of subchannels in coded multichannel ALOHA for any initial rate.**

***Index Terms*— Multichannel ALOHA, incremental redundancy, throughput analysis.**

I. Introduction

THERE has been growing interest in machine type communication (MTC) [1], [2]to support the connectivity for a number of devices and sensors, which might be important in the Internet of Things (IoT). For MTC, various random access schemes are considered for low signaling overhead as in [3]–[5]. Based on multichannel ALOHA in [6] and [7], random access for MTC has also been studied in [8] and [9]. As shown in [8], multichannel ALOHA has a scaling property which might be suitable for MTC with a number of devices.

To understand the performance of multichannel ALOHA in terms of throughput, a simple collision model [10] is widely used. Although the simple collision model allows a tractable analysis, it is not useful if coded packets [11], [4] are transmitted for better performance under an interference-limited environment. Thus, we need to consider a different approach that can take into account more realistic channel environments as well as performance behaviors of channel coding. From this point of view, the approach proposed in [11] is appealing, which is an information-theoretic approach. According to this approach, although multiple packets collide, a receiver can extract some useful information that can be used to combine with re-transmitted packets for decoding.

In [12], the notion of coded ALOHA is introduced and a graph-based analysis is considered for performance analysis. In [13], a broadcast protocol is studied based on the approach in [12]. While the graph-based analysis approach can be carried out for a given specific channel coding scheme, it is not easy to accommodate channel conditions (e.g., fading). Thus, in this letter, we adopt the information-theoretic approach proposed in [11] to understand the performance of multichannel ALOHA when coded packets are transmitted under Rayleigh fading. A closed-form expression for the throughput[1] is derived, which is the main contribution of this letter. The scaling property of multichannel ALOHA is also clearly shown by the closed-form expression in terms of limiting throughput.

Notation: Matrices and vectors are denoted by upper- and lower-case boldface letters, respectively. $\mathbf{I}$ stands for the identity matrix. $\mathbb{E}[\cdot]$ denotes the statistical expectation. $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ represents the distribution of circularly symmetric complex Gaussian (CSCG) random vector with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{R}$.

II. System Model

We consider a random access system consisting of a base station (BS) and a number of users for uplink transmissions. For multichannel ALOHA, we assume that there are L orthogonal subchannels and an active user with a data packet can randomly choose a subchannel. Let $h_{k,l;t}$ denote the channel coefficient (of block-fading channel) of the lth subchannel from the kth user who chooses at time slot t. Then, the received signal at the BS is given by

$$\mathbf{r}_{l;t} = \sum_{k \in I_{l;t}} h_{k,l;t} \sqrt{P} \mathbf{s}_{k;t} + \mathbf{n}_{l;t}, \tag{1}$$

where $I_{l;t}$ stands for the index set of the active users who choose the lth subchannel at time slot t, $\mathbf{s}_{k;t}$ is the (zero-mean) coded subblock transmitted by the kth active user at time slot t, and $\mathbf{n}_{l;t} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ is the background noise. Here, P is the transmission power.

Let $\mathcal{T}_k$ stand for the index set of time slots when the kth user becomes active and transmits a coded subblock. Throughout this letter, we assume that $\mathbf{s}_{k;t}$, $t \in \mathcal{T}_k$, is a subblock of coded sequence and a Gaussian codebook is used for encoding to achieve the capacity [11]. Thus, $\mathbf{s}_{k;t}$, $t \in \mathcal{T}_k$, can be seen as a CSCG random vector with $\mathbb{E}[\mathbf{s}_{k;t}] = \mathbf{0}$ and $\mathbb{E}[\mathbf{s}_{k;t}\mathbf{s}_{k;t}^{\mathrm{H}}] = \mathbf{I}$.

Let $\alpha_{k,l;t} = |h_{k,l;t}|^2$. If user k is the target user and active to transmit a subblock over subchannel l at time slot t (i.e., $k \in I_{l;t}$), Eq. (1) can be re-written as

$$\mathbf{r}_{l;t} = \sqrt{P} h_{k,l;t} \mathbf{s}_{k;t} + \mathbf{w}_{l;t}, \tag{2}$$

where $\mathbf{w}_{l;t} \sim \mathcal{CN}(\mathbf{0}, \sigma_{k,l;t}^2 \mathbf{I})$. Here, $\sigma_{k,l;t}^2$ is given by

$$\sigma_{k,l;t}^2 = P \sum_{m \in I_{l;t} \setminus k} \alpha_{m,l;t} + N_0. \tag{3}$$

Then, the instantaneous achievable rate (in bits/Hz) becomes

$$C_{k,l;t} = \log_2 \left(1 + \frac{P \alpha_{k,l;t}}{\sigma_{k,l;t}^2} \right). \tag{4}$$

III. Throughput Analysis

In this section, we derive a closed-form expression for the throughput of coded multichannel ALOHA under independent Rayleigh fading.

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[1]In this letter, the throughput is defined as the number of (successfully transmitted) bits per Hz.

A. Throughput via Number of Re-Transmissions

As in [11], a user can employ incremental redundancy (IR) for a re-transmission strategy or protocol. For convenience, this strategy is referred to as the incremental redundancy protocol (IRP). In this case, we can assume that each subblock is a subsequence of a long coded sequence and a user transmits a different subblock when the user becomes active with access probability p_a, which is a design parameter. Then, based on IR, the accumulated achievable rate becomes

$$R_k(\mathcal{T}_k) = \sum_{t \in \mathcal{T}_k} C_{k,t}, \tag{5}$$

where $C_{k,t} = \sum_{l=1}^{L} C_{k,l;t}$. Denoting by $l_k\ (t)$ the index of the subchannel that user k chooses at time t, we have $C_{k,t} = C_{k,l_k(t);t}$. Note that if user k is not active at time t, $C_{k,l_k(t);t} = 0$.

In IRP, a message sequence is encoded at an initial rate, denoted by R. Then, the number of re-transmissions for a successful recovery of a message sequence is given by

$$\tau_k = \min\{|\mathcal{T}_k| : \ R_k(\mathcal{T}_k) \geq R\}. \tag{6}$$

Noting that τ_k is a stopping time [14], we can show that

$$\mathbb{E}[\tau_k]\mathbb{E}[C_{k,t}] \geq R \text{ and } (\mathbb{E}[\tau_k] - 1)\mathbb{E}[C_{k,t}] < R. \tag{7}$$

Then, the average number of re-transmissions is bounded as

$$\frac{R}{\mathbb{E}[C_{k,t}]} \leq \mathbb{E}[\tau_k] < \frac{R}{\mathbb{E}[C_{k,t}]} + 1. \tag{8}$$

From the average number of re-transmissions, the average transmission delay becomes $D = \frac{\mathbb{E}[\tau_k]}{p_a}$ and the throughput (per user) can be expressed as a function of the average number of re-transmissions as follows:

$$\eta = \frac{R}{D} = \frac{R p_a}{\mathbb{E}[\tau_k]}. \tag{9}$$

From (8), we can also show that

$$\frac{p_a \mathbb{E}[C_{k,t}]}{1 + \frac{\mathbb{E}[C_{k,t}]}{R}} < \eta \leq p_a \mathbb{E}[C_{k,t}]. \tag{10}$$

For a large R, the two bounds in (10) are close to each other. It is interesting to note that the throughput increases with p_a as shown in (10) (at the cost of a longer delay), which is different from conventional ALOHA where the throughput approaches 0 as $p_a \to 1$. This interesting behavior was found in [11].

B. Average Achievable Rate

According to (7), we may need a closed-form expression for $\mathbb{E}[C_{k,t}]$ to determine the average number of re-transmissions, $\mathbb{E}[\tau_k]$, or its bounds, which allows to find a closed-form expression for the throughput. In this subsection, we derive a closed-form expression[2] for $\mathbb{E}[C_{k,t}]$ when K is large with fixed $K p_a / L$ under the following assumption on the channels.

A) The channel coefficients are independent CSCG random variables, i.e., $h_{k,l;t} \sim \mathcal{CN}(0, \sigma_h^2)$ (i.e., we assume independent Rayleigh fading channels).

[2]This is the main contribution of this letter. Note that no closed-form expressions for the throughput are obtained in [11].

Then, the $\alpha_{k,l;t}$'s are iid exponential random variables under the assumption of **A**.

Suppose that k becomes an element of $\mathcal{I}_{l;t}$, i.e., user k becomes active at time slot t. In this case, if there are q other active users choosing the same subchannel as user k, i.e., $|\mathcal{I}_{l;t} \setminus k| = q$, the statistical properties of $C_{k,l_k\ (t);t}$ are the same as the following quantity:

$$Z(q) = \log_2\left(1 + \frac{P\alpha_1}{P\sum_{m=1}^{q} \alpha_{m+1} + N_0}\right), \tag{11}$$

where the α_q's are iid. Thus, in order to find $\mathbb{E}[C_{k,l_k\ (t);t}] = \mathbb{E}[C_{k,t}]$, we can use $Z(q)$ instead of $C_{k,l_k\ (t);t}$ itself.

Lemma 1: For any positive integer $n \geq 1$, we have

$$\mathbb{E}[Z(Q)] \geq \underline{Z}_n = \sum_{q=0}^{n-1} \bar{Z}(q) \Pr(Q = q) + \Pi_1\left(\frac{P\sigma_h^2}{N_n}\right) \Pr(Q \geq n), \tag{12}$$

where $\bar{Z}(q) = \Pi_{q+1}\left(\frac{P\sigma_h^2}{N_0}\right) - \Pi_q\left(\frac{P\sigma_h^2}{N_0}\right)$. Here, $\Pi_q(x) = \frac{1}{\ln 2} e^{1/x} \sum_{n=1}^{q} E_n(1/x)$ and $E_n\ (x) = \int_1^\infty t^{-n} e^{-tx} dt$. Note that $\Pi_0(x) = 0$. In addition, $N_n = P\sigma_h^2 \mathbb{E}[Q \mid Q \geq n] + N_0$, where

$$\mathbb{E}[Q \mid Q \geq n] = \frac{\mathbb{E}[Q] - \sum_{q=0}^{n-1} q \Pr(Q = q)}{\Pr(Q \geq n)}. \tag{13}$$

As $n \to \infty$, the lower-bound in (12) becomes tighter and approaches $\mathbb{E}[Z(Q)]$.

Proof: Letting $\bar{Z}(q) = \mathbb{E}[Z(q)]$, it can be shown that

$$\mathbb{E}[Z(Q)] = \sum_{q=0}^{n-1} \bar{Z}(q) \Pr(Q = q) + \mathbb{E}[Z(Q) | Q \geq n] \Pr(Q \geq n). \tag{14}$$

Using Jensen's inequality, we can show that

$$\begin{aligned} &\mathbb{E}[Z(Q) | Q \geq n] \\ &\geq \mathbb{E}_{\alpha_1}\left[\log_2\left(1 + \frac{P\alpha_1}{\mathbb{E}[Px \sum_{m=1}^{Q} \alpha_{m+1} | Q \geq n] + N_0}\right)\right] \\ &= \mathbb{E}_{\alpha_1}\left[\log_2\left(1 + \frac{P\alpha_1}{N_n}\right)\right] = \Pi_1\left(\frac{P\sigma_h^2}{N_n}\right). \end{aligned} \tag{15}$$

According to [15] [16], it can be shown that

$$\begin{aligned} \bar{Z}(q) &= \mathbb{E}\left[\log_2\left(N_0\left(1 + \frac{P\sum_{m=1}^{q+1} \alpha_m}{N_0}\right)\right)\right] \\ &\quad - \mathbb{E}\left[\log_2\left(N_0\left(1 + \frac{P\sum_{m=1}^{q} \alpha_{m+1}}{N_0}\right)\right)\right] \\ &= \Pi_{q+1}\left(\frac{P\sigma_h^2}{N_0}\right) - \Pi_q\left(\frac{P\sigma_h^2}{N_0}\right). \end{aligned} \tag{16}$$

Substituting (16) and (15) into (14), we can show (12).

It is noteworthy that even if the $h_{k,l;t}$'s are not CSCG random variables, the result in (12) is valid if Π_q is replaced with $\mathbb{E}\left[\log_2\left(1 + \frac{P\sum_{m=1}^{q} \alpha_{m+1}}{N_0}\right)\right]$. For example, if the additive white Gaussian noise (AWGN) channel is considered, $\alpha_m = 1$. In this case, $\Pi_q = \log_2\left(1 + \frac{Pq}{N_0}\right)$.

We also note that as $n \to \infty$, the bound in (12) approaches the exact one. Thus, a tighter bound can be achieved for a larger n. ■

We now assume that Q is a Poisson random variable with parameter $\lambda = Kp_a/L$, which is valid as $K \to \infty$ with λ converging to a constant. Then, for a finite n, we are able to have a closed-form expression for $\mathbb{E}[Z(Q)]$ with a finite number of terms. For example, if $n = 2$, we have

$$\begin{aligned}\underline{Z}_2 = &\, Pi_2(\gamma)e^{-\lambda}\lambda + \Pi_1(\gamma)e^{-\lambda}(1-\lambda) \\ &+\Pi_1(\gamma_2)(1 - e^{-\lambda}(1+\lambda)),\end{aligned}$$

where $\gamma = \frac{P\sigma_h^2}{N_0}$ is the signal-to-noise ratio (SNR) and $\gamma_2 = \frac{P\sigma_h^2}{P\mathbb{E}[Q|Q\geq 2]+N_0}$. Here, $\mathbb{E}[Q|Q \geq 2] = \frac{\lambda - \lambda e^{-\lambda}}{1-(1+\lambda)e^{-\lambda}}$.

Using $\underline{Z}_n$ with a sufficiently large n, a lower-bound on the (system) throughput becomes

$$T_{\text{sys}} = K\eta \geq \tilde{T}_n = \frac{Kp_a \underline{Z}_n}{1 + \frac{\underline{Z}_n}{R}} = \frac{L\lambda \underline{Z}_n}{1 + \frac{\underline{Z}_n}{R}}. \quad (17)$$

The closed-form expression in (17) allows us to optimize key parameters such as the access probability, p_a. Note that the lower-bound in (17) becomes an approximation if Q is a binomial random variable.

From (17), we can have an interesting asymptotic[3] behavior as follows.

Lemma 2: Suppose that Q is a Poisson random and the $\alpha_{k,l;t}$'s are iid and $\mathbb{E}[|h_{k,l;t}|^2] = \mathbb{E}[\alpha_{k,l;t}] < \infty$ (not necessarily Rayleigh fading channels). Then, for any $R > 0$, the asymptotic system throughput (the total number of successfully transmitted bits per Hz) grows linearly with L as

$$\lim_{\lambda\to\infty} T_{\text{sys}} = \frac{L}{\ln 2}. \quad (18)$$

Proof: From (12), as $\lambda \to \infty$, the first n terms approach 0 and $\Pr(Q \geq n)$ approaches 1 for a finite n. Thus, $\lim_{\lambda\to\infty} \underline{Z}_n = \lim_{\lambda\to\infty} \Pi_1\left(\frac{P\sigma_h^2}{N_n}\right)$. Since $N_n \to \infty$ as $\lambda \to \infty$, we have $\lim_{\lambda\to\infty} \underline{Z}_n = 0$. This indicates that the denominator in (17) is 1 when $\lambda \to \infty$ for any $R > 0$.

We now consider

$$\lambda \underline{Z}_n = \sum_{q=0}^{n-1} \bar{Z}(q)\lambda \Pr(Q = q) + \lambda \Pi_1\left(\frac{P\sigma_h^2}{N_n}\right)\Pr(Q \geq n).$$

Similarly, as $\lambda \to \infty$, the first n terms approach 0 and $\Pr(Q \geq n)$ approaches 1 for a finite n. Thus,

$$\begin{aligned}\lim_{\lambda\to\infty} \lambda\underline{Z}_n &= \lim_{\lambda\to\infty} \lambda\Pi_1\left(\frac{P\sigma_h^2}{N_n}\right) \\ &= \lim_{\lambda\to\infty} \lambda\mathbb{E}\left[\log_2\left(1 + \frac{P\alpha_1}{N_n}\right)\right]. \quad (19)\end{aligned}$$

[3]The same result under slightly different conditions for $L = 1$ can be found in [11, Eq. (30)]. The main difference is that the result in this letter does not require optimal R (in [11], an optimal R is required). That is, the result is valid for any $R > 0$.

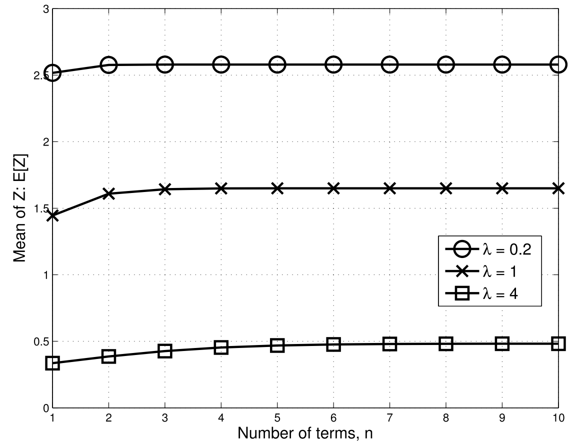

Fig. 1. $\underline{Z}_n$ for different numbers of n with $\gamma = 10$ dB.

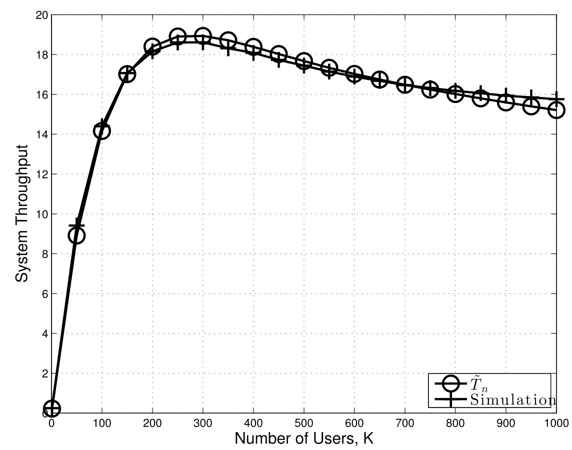

Fig. 2. System throughput for various values of the number of users, K, when $\gamma = 10$ dB, $p_a = 0.1$, $R = 10$, and $L = 10$.

Since N_n increases with λ, for a large λ, we have

$$\mathbb{E}\left[\log_2\left(1 + \frac{P\alpha_1}{N_n}\right)\right] \approx \frac{1}{\ln 2}\mathbb{E}\left[\frac{P\alpha_1}{N_n}\right] = \frac{P\sigma_h^2}{N_n \ln 2}. \quad (20)$$

In addition, for a large λ, we have $\mathbb{E}[Q|Q \geq n] = \mathbb{E}[Q] = \lambda$ (for a finite n). Thus,

$$\begin{aligned}\lim_{\lambda\to\infty} \lambda\mathbb{E}\left[\log_2\left(1 + \frac{P\alpha_1}{N_n}\right)\right] &= \lim_{\lambda\to\infty} \lambda \frac{P\sigma_h^2}{(\lambda P\sigma_h^2 + N_0)\ln 2} \\ &= \frac{1}{\ln 2}. \quad (21)\end{aligned}$$

From this and the fact that $\lim_{\lambda\to\infty} \underline{Z}_n \to 0$, we can show (18) with the inequality ($\geq$) rather than the equality. Since $\tilde{T}_n$ approaches $K\eta$ as $R \to \infty$ and $n \to \infty$, (18) is valid with the equality. ■

IV. Simulation Results

In this section, we present simulation results with the channels that are generated according to the assumption of **A**. For convenience, we assume that $\sigma_h^2 = 1$ and $N_0 = 1$.

We need to verify the impact of the number of terms, n, on $\underline{Z}_n$. With $\gamma = 10$ dB, we can show $\underline{Z}_n$ for different numbers of n in Fig. 1. The lower-bound $\underline{Z}_n$ increases with n,

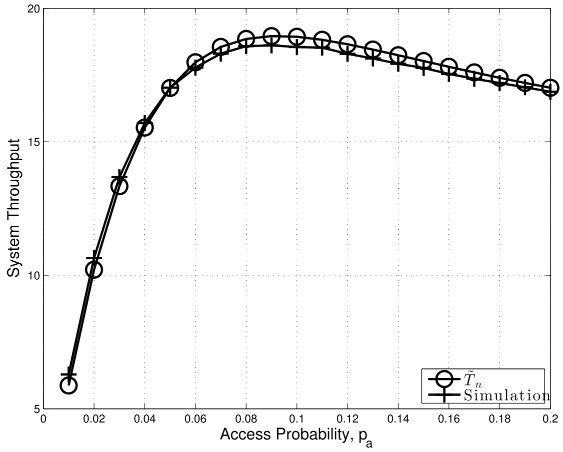

Fig. 3. System throughput for various values of access probability, p_a, when $\gamma = 10$ dB, $K = 300$, $R = 10$, and $L = 10$.

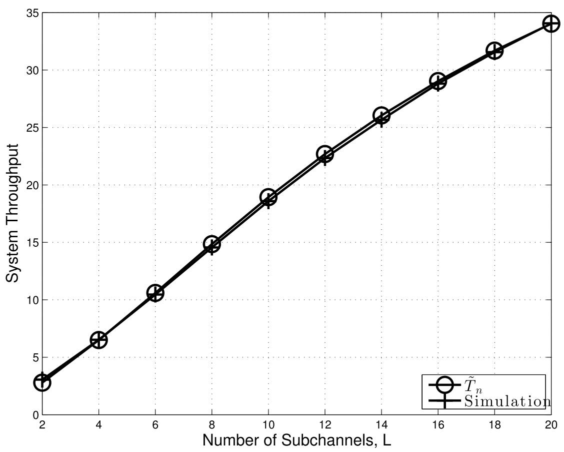

Fig. 4. System throughput for various values of the number of subchannels, L, when $\gamma = 10$ dB, $p_a = 0.1$, $R = 10$, and $K = 300$.

and with $n \geq 8$, it can converge (for various values of λ). Thus, it is possible to have a tight bound on or a good approximation of the throughput using $\underline{Z}_n$ with $n \geq 8$. For the rest of simulations, n is set to 10.

Fig. 2 shows the system throughput for various values of the number of users, K, when $\gamma = 10$ dB, $p_a = 0.1$, $R = 10$, and $L = 10$. As the number of users increases, the system throughput increases and then decreases. However, as shown in Lemma 2, the system throughput does not approach 0, but $\frac{L}{\ln 2} = 14.42$ as K (or λ) increases (with a finite R).

Fig. 3 shows the system throughput for various values of access probability, p_a, when $\gamma = 10$ dB, $K = 300$, $R = 10$, and $L = 10$. The performance behavior as p_a increases in Fig. 3 is similar to that in Fig. 2. We note that since the system throughput $\tilde{T}_n$ is obtained under the assumption that Q is a Poisson random variable, it is an approximation of T_{sys} when Q is a binomial random variable as in this simulation.

In order to see the impact of the number of subchannels, L on the performance, we show the system throughput for various values of L in Fig 4 when $\gamma = 10$ dB, $p_a = 0.1$, $R = 10$, and $K = 300$. It is clearly shown that the system throughput increases with L.

V. Conclusions

Based on an information-theoretic approach, we derived a closed-form expression for the throughput of coded multichannel ALOHA when the number of users is large. Although the derived expression was a lower-bound, it can be arbitrarily tight by including more terms. It was also shown that the asymptotic system throughput grows linearly with the number of subchannels.

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