# Single-Carrier Index Modulation and CS Detection

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***Abstract*—We consider index modulation (IM) for single-carrier (SC) systems in this paper. Compared with conventional orthogonal frequency division multiplexing (OFDM) IM, the resulting approach, which is referred to as SCIM, has a better performance due to the path diversity gain under a multipath fading environment. For sparse IM, since compressive sensing (CS) algorithms can be used for low-complexity signal detection, we consider the orthogonal matching pursuit (OMP) algorithm to perform the signal detection in SCIM. A transmit diversity scheme is also proposed for both OFDM-IM and SCIM, which can not only improve the performance in terms of the diversity gain, but also increase the number of information bits per signal block.**

***Index Terms*—sparsity; index modulation; compressive sensing; diversity**

## I. Introduction

Orthogonal frequency division multiplexing (OFDM) with index modulation (IM) has been proposed in [1], where a subset of subcarriers are active and the indices of them are also used to convey information bits. An overview of IM techniques is presented in [2]. In [3], a performance analysis is carried out when the maximum likelihood (ML) detector is employed.

In [1], the set of subcarriers is divided into multiple subsets or clusters and in each subset both IM and conventional modulation such as quadrature amplitude modulation (QAM) are employed to transmit information bits. In general, the number of subcarriers in each subset is not large in order to avoid a high computational complexity for the ML detection. For example, if there are 128 subcarriers, the number of subsets is 8 and the number of subcarriers per subset is 16. If there are 8 subcarriers are active in each subset the number of bits to be transmitted by IM becomes $8\lfloor \log_2 \binom{16}{8} \rfloor = 104$ bits, while the complexity to perform the ML detection is proportional to $8 \times 2^{13} = 2^{16}$ if an exhaustive search is used. On the other hand, if the number of subsets becomes 16 and a half of the subcarriers in each subset become active for IM, the number of bits to be transmitted by IM becomes $16\lfloor \log_2 \binom{8}{4} \rfloor = 96$ bits and the complexity to perform the ML detection is proportional to $16 \times 2^6 = 2^{10}$. Clearly, this example demonstrates that there is a trade-off between the complexity of ML detection and the number of information bits transmitted by IM through the number of subsets.

In [4], sparse IM is considered in order to employ a low-complexity detector, while it can also be used for multiple access [5] [6]. Due to the sparsity of active subcarriers, the notion of compressive sensing (CS) [7] [8] can be exploited to derive low-complexity detection methods. In Fig. 1, we show numerical results to compare the number of information bits and the detection complexity of OFDM-IM and sparse IM when the number of subcarriers is 256 and 4-QAM is used for each active subcarrier. In OFDM-IM, the number of information bits and the detection complexity (based on the ML approach) vary depending on the number of subsets. In particular, as mentioned earlier, we can see that the number of information bits and the detection complexity decrease with the number of subsets. In sparse IM, it is assumed that 51 subcarriers are active among 256 subcarriers. For the low-complexity CS based detector, we assume that the orthogonal matching pursuit (OMP) algorithm [9], [10] is used, which has the complexity proportional to the square of the number of subcarriers. From Fig. 1, it is clear that sparse IM can have a more number of information bits with a lower complexity for the detection than OFDM-IM with $2^4$ subsets.

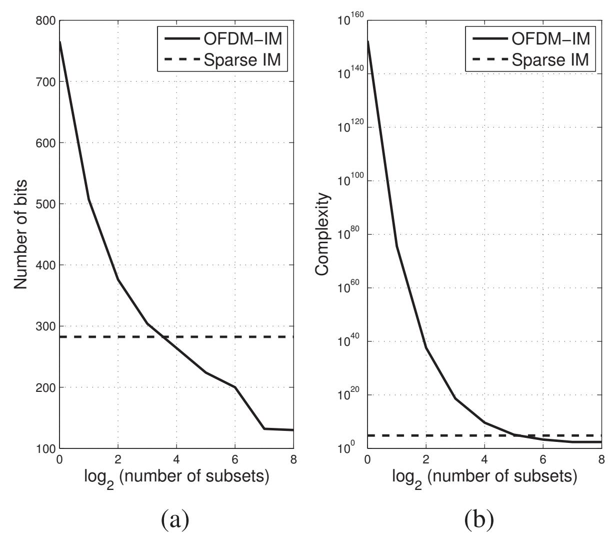


Fig. 1. Comparison between OFDM-IM and sparse IM when the number of subcarriers is 256: (a) the number of information bits; (b) detection complexity.

Although there are a number of advantages of OFDM (e.g., effective mitigation of inter-symbol interference (ISI), low-complexity one-tap equalization, and so on), OFDM has also various drawbacks [11] [12] [13]. For example, OFDM cannot exploit the path diversity gain for uncoded signals, which is also true for OFDM-IM. If some active subcarriers for IM

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experience deep fading, the receiver cannot detect them, which results in a poor performance. In this paper, we consider IM with a single-carrier (SC) system that is proposed in [11]. There are a few advantages of SC system over multicarrier (MC) system including the path diversity gain for uncoded signals. The resulting IM can have the path diversity gain and performs better than OFDM-IM. In addition, we consider sparse IM so that low-complexity CS algorithms can be used for the signal detection.

We also propose a transmit diversity scheme that can be applied to IM for both SC and MC systems. This scheme can not only improve the diversity gain, but also increase the number of bits per signal block. Since the error probability can be low due to the diversity gain, the resulting scheme could be used for reliable transmissions without relying on channel coding when transmitters have limitations in terms of hardware complexity. Thus, the resulting scheme is well-suited to sensors in wireless sensor networks (WSNs) or small devices in the Internet of Things (IoT) [14] [15].

It is noteworthy that the application of IM to SC systems has been studied in [16], which was pointed out by one of reviewers. The main differences from [16] are as follows: *i)* we focus on sparse IM that allows to use CS algorithms for signal detection; *ii)* a transmit diversity scheme is proposed, which can be used in both SC and MC systems.

*Notation*: Matrices and vectors are denoted by upper- and lower-case boldface letters, respectively. The superscripts $\mathrm{T}$ and $\mathrm{H}$ denote the transpose and complex conjugate, respectively. The $p$-norm of a vector $\mathbf{a}$ is denoted by $||\mathbf{a}||_p$ (If $p = 2$, the norm is denoted by $||\mathbf{a}||$ without the subscript). The superscript $\dagger$ denotes the pseudo-inverse. For a vector $\mathbf{a}$, $\mathrm{diag}(\mathbf{a})$ is the diagonal matrix with the diagonal elements from $\mathbf{a}$. For a matrix $\mathbf{X}$ (a vector $\mathbf{a}$), $[\mathbf{X}]_n$ ($[\mathbf{a}]_n$) represents the $n$th column (element, resp.). If $n$ is a set of indices, $[\mathbf{X}]_n$ is a submatrix of $\mathbf{X}$ obtained by taking the corresponding columns. The Kronecker product is denoted by $\otimes$. $\mathbb{E}[\cdot]$ and $\mathrm{Var}(\cdot)$ denote the statistical expectation and variance, respectively. $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ ($\mathcal{N}(\mathbf{a}, \mathbf{R})$) represents the distribution of circularly symmetric complex Gaussian (CSCG) (resp., real-valued Gaussian) random vectors with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{R}$.

## II. System Model

We consider SC transmission over an ISI channel with cyclic prefix (CP) [11]. Let $\mathbf{s} = [s_0 \ \ldots \ s_{L-1}]^{\mathrm{T}}$ denote a block of data symbols to be transmitted over an ISI channel, where $L$ is the length of $\mathbf{s}$. Then, the received signal at time $l$ is given by

$$r_l = \sum_{p=0}^{P-1} h_p s_{l-p} + n_l, \quad (1)$$

where $h_p$ is the $p$th coefficient of the ISI channel of length $P$ and $n_l \sim \mathcal{CN}(0, N_0)$ is the background noise. For the transmission of each block, a CP is appended to $\mathbf{s}$. At the receiver after removing the signal corresponding to CP, we have

$$\begin{aligned}\mathbf{r} &= [r_0 \ \ldots \ r_{L-1}]^{\mathrm{T}} \\ &= \mathbf{H}_{\mathrm{isi}}\mathbf{s} + \mathbf{n}, \end{aligned} \quad (2)$$

where $\mathbf{n} = [n_0 \ \ldots \ n_{L-1}]^{\mathrm{T}}$ and $\mathbf{H}_{\mathrm{isi}}$ is a cyclic matrix that is given by

$$\mathbf{H}_{\mathrm{isi}} = \begin{bmatrix} h_0 & h_{L-1} & \cdots & h_1 \\ h_1 & h_0 & \cdots & h_2 \\ \vdots & \vdots & \ddots & \vdots \\ h_{L-1} & h_{L-2} & \cdots & h_0 \end{bmatrix},$$

where $h_P = \ldots = h_{L-1} = 0$ for $L > P$. For IM, we assume that $\mathbf{s}$ is $Q$-sparse, i.e., $\mathbf{s} \in \Sigma_Q$, where

$$\Sigma_Q = \{\mathbf{s} \,|\, ||\mathbf{s}||_0 \le Q\}.$$

In addition, we assume that non-zero elements of $\mathbf{s}$ has an element of an $M$-ary constellation, i.e., $s_l \in \mathcal{S}$ if $s_l \neq 0$, where $\mathcal{S}$ is the signal constellation and $|\mathcal{S}| = M$. In addition, we assume that zero is not an element of $\mathcal{S}$, i.e., $0 \notin \mathcal{S}$. For convenience, we assume that a non-zero element of $\mathbf{s}$, i.e., $s_l \in \mathcal{S}$, has the following properties:

$$\begin{aligned}\mathbb{E}[s_l] &= 0 \\ \mathrm{Var}(s_l) &= \sigma_s^2.\end{aligned}$$

For example, if we consider binary phase shift keying (BPSK) for $\mathcal{S}$ with $\mathcal{S} = \{-A, A\}$, we have $\sigma_s^2 = A^2$. Then, the number of information bits per signal block becomes

$$N_{\mathrm{b}} = \lfloor \log_2 \binom{L}{Q} \rfloor + Q \log_2 M. \quad (3)$$

The resulting system is referred to as single-carrier IM or SCIM in this paper. SCIM can be seen as a time-domain version of IM with single cluster in [1] or a generalization of pulse-position modulation (PPM). To see that PPM is a special case of SCIM, we can assume that $s_l = \{A, 0\}$ and $Q = 1$, which becomes a $Q$-ary PPM.

Note that the number of information bits transmitted by IM in (3) can be maximized if $Q = L/2$ for an even $L$. Unfortunately, in this case, the complexity of the ML signal detection can be high if an exhaustive search is used.

## III. Low-Complexity Detection Algorithms

In this section, we discuss low-complexity detection algorithms for SCIM based on existing approaches.

### A. ML and MMSE Detection

In order to estimate $\mathbf{s}$, we can consider the ML approach. The ML estimate can be found as [17]

$$\begin{aligned}\hat{\mathbf{s}} &= \underset{\mathbf{s} \in \bar{\Sigma}_Q}{\mathrm{argmax}}\, f(\mathbf{r} \,|\, \mathbf{s}) \\ &= \underset{\mathbf{s} \in \bar{\Sigma}_Q}{\mathrm{argmin}}\, ||\mathbf{r} - \mathbf{H}_{\mathrm{isi}}\mathbf{s}||^2, \end{aligned} \quad (4)$$

where

$$\bar{\Sigma}_Q = \{\mathbf{s} \,|\, \mathbf{s} \in \Sigma_Q, \ s_l \in \mathcal{S} \cup \{0\}\}. \quad (5)$$

We note that $|\bar{\Sigma}_Q| = 2^{N_b}$, which grows exponentially with $Q$. Since the complexity of the ML detection is proportional to $|\bar{\Sigma}_Q|$, if an exhaustive search is used, it might be computationally prohibitive unless $Q$ is small (i.e., $Q$ is 1 or 2) as mentioned earlier.

As in [11], [12], the frequency domain equalization (FDE) can be considered to detect $\mathbf{s}$ with low-complexity. To this end, we can apply the discrete Fourier transform (DFT) to $\mathbf{r}$ in (2). After DFT, we have

$$\begin{aligned} \mathbf{y} &= \mathbf{F}\mathbf{r} \\ &= \mathbf{F}\mathbf{H}_{\rm isi}\mathbf{s} + \mathbf{F}\mathbf{n} \\ &= \mathbf{F}\mathbf{H}_{\rm isi}\mathbf{F}^{\rm H}\mathbf{F}\mathbf{s} + \mathbf{F}\mathbf{n} \\ &= \mathbf{H}\mathbf{F}\mathbf{s} + \tilde{\mathbf{n}}, \end{aligned} \tag{6}$$

where $\tilde{\mathbf{n}} = \mathbf{F}\mathbf{n}$, $\mathbf{F}$ is the DFT matrix, and $\mathbf{H}$ is a diagonal matrix, which is referred to as the frequency-domain channel matrix and given by

$$\mathbf{H} = \mathrm{diag}(H_0, \ldots, H_{L-1}).$$

Here, $H_l = \sum_{p=0}^{P-1} h_p e^{-\frac{j2\pi pl}{L}}$. The DFT matrix, $\mathbf{F}$, is given by

$$[\mathbf{F}]_{m,l} = \frac{1}{\sqrt{L}} e^{-\frac{j2\pi ml}{L}},\ m, l = 0, \ldots, L-1.$$

In FDE, we estimate $\mathbf{x} = \mathbf{F}\mathbf{s}$ (instead of $\mathbf{s}$) using the minimum mean squared error (MMSE) filter (which is a single-tap equalizer) that is given by

$$\begin{aligned} \mathbf{W}_{\rm mmse} &= \mathbb{E}[\mathbf{x}\mathbf{y}^{\rm H}] \left(\mathbb{E}[\mathbf{y}\mathbf{y}^{\rm H}]\right)^{-1} \\ &= \mathbf{H}^{\rm H} \left(\mathbf{H}\mathbf{H}^{\rm H} + \frac{1}{Q\gamma}\mathbf{I}\right)^{-1} \\ &= \mathrm{diag}\left(\frac{H_0^*}{|H_0|^2 + \frac{1}{Q\gamma}}, \ldots, \frac{H_{L-1}^*}{|H_{L-1}|^2 + \frac{1}{Q\gamma}}\right), \end{aligned} \tag{7}$$

where $\gamma = \frac{\sigma_s^2}{N_0}$. Note that in (7), we assume that the non-zero elements of $\mathbf{s}$ are uniformly distributed. In this case, we have

$$\mathbb{E}[\mathbf{x}\mathbf{x}^{\rm H}] = \mathbb{E}[\mathbf{s}\mathbf{s}^{\rm H}] = Q\sigma_s^2\mathbf{I}.$$

Once $\mathbf{x}$ is estimated as $\mathbf{W}_{\rm mmse}\mathbf{y}$, $\mathbf{s}$ can be recovered by taking inverse DFT (IDFT). That is,

$$\begin{aligned} \hat{\mathbf{s}}_{\rm mmse} &= \mathbf{F}^{-1}\mathbf{W}_{\rm mmse}\mathbf{y} \\ &= \mathbf{F}^{\rm H}\mathbf{W}_{\rm mmse}\mathbf{y}. \end{aligned} \tag{8}$$

From the estimate of $\mathbf{s}$ in (8), the largest $Q$ elements in terms of their amplitudes can be chosen for the detection of index modulated signals.

### B. Low-Complexity CS Detection

In this subsection, another low-complexity detection method is derived using CS algorithms to exploit the sparsity of $\mathbf{s}$ as in [4].

Consider the ML detection problem in (4). Note that since the size of $\mathbf{H}_{\rm isi}$ is $L \times L$, an estimate of $\mathbf{s}$ can be obtained by using the inverse of $\mathbf{H}_{\rm isi}$ (which results in the least squares (LS) solution). However, this requires a high computational complexity. Thus, based on the notion of CS, various low-complexity approaches can be used to estimate $Q$-sparse $\mathbf{s}$. In the context of CS, the estimation of $\mathbf{s}$ from (2) can be seen as the following problem:

$$\text{Find } Q\text{-sparse } \mathbf{s} \text{ such that } \mathbf{r} \approx \mathbf{H}_{\rm isi}\mathbf{s},$$

where $\mathbf{H}_{\rm isi}$ becomes the measurement matrix. For example, the OMP algorithm in [9], [10] can be used. In this paper, we consider a detection method that uses the OMP algorithm and call it the CS detector for convenience. In general, the computational complexity of the OMP algorithm to find $Q$-sparse $\mathbf{s}$ from $\mathbf{r}$ depends on the size of the measurement matrix and sparsity [18]. In this case, provided that $Q$ is sufficiently small, the computational complexity becomes $O(L^2)$

To reduce the computational complexity of the CS detector, we may use the property of the frequency-domain channel matrix $\mathbf{H}$. Noting that $\mathbf{H}$ is diagonal and the amplitudes of the diagonal elements are different, we can use a subset of the received signals of $\mathbf{y}$ that correspond to the $N$ largest amplitudes of the diagonal elements of $\mathbf{H}$, where $N \le L$. In this case, the complexity of the OMP algorithm can be lowered as $O(NL)$ with a slightly degraded performance as the received signals of the lowest channel gains are not used. To this end, let

$$|H_{l(0)}| \ge \ldots \ge |H_{l(N-1)}| \ge \ldots \ge |H_{l(L-1)}| \tag{9}$$

and $\mathcal{B} = \{l(0), \ldots, l(N-1)\}$, where $l(n)$ denotes the index of the $n$th largest element of $\{H_l\}$ in terms of its amplitude. In addition, let $\mathbf{y}_{\mathcal{B}}$ and $\tilde{\mathbf{n}}_{\mathcal{B}}$ denote the subvectors of $\mathbf{y}$ and $\tilde{\mathbf{n}}$, respectively, obtained by taking the elements corresponding to $\mathcal{B}$. Then, from (6), we have

$$\mathbf{y}_{\mathcal{B}} = \mathbf{A}\mathbf{s} + \tilde{\mathbf{n}}_{\mathcal{B}}, \tag{10}$$

where $\mathbf{A}$ (of size $N \times L$) is the submatrix of $\mathbf{HF}$ that is obtained by taking the rows corresponding to $\mathcal{B}$. Note that $N$ cannot be too small although the noise can be negligible in order to have a reasonable performance according to recovery guarantee conditions [18].

In general, in the CS detector, the performance is dominated by the detection performance of the support of $\mathbf{s}$. If the support of $\mathbf{s}$ is erroneously estimated, the data symbols of the active elements of $\mathbf{s}$ cannot be correctly detected. Thus, in Section IV, we consider a transmit diversity scheme for more reliably transmission of information bits by IM, while we only consider the detection performance of the support of $\mathbf{s}$ in Section V.

## IV. A Transmit Diversity Scheme

In this section, we propose a transmit diversity scheme for IM based on precoding in order to improve the detection performance for the information bits transmitted by IM.

### A. Superposition of Precoded Signals

Suppose that $\mathbf{s}$ is a superposition of $D$ sparse signals as follows:

$$\mathbf{s} = \sum_{d=0}^{D-1} \boldsymbol{\Psi}_d \mathbf{s}_d = [\boldsymbol{\Psi}_0 \ \dots \ \boldsymbol{\Psi}_{D-1}][\mathbf{s}_0^{\mathrm{T}} \ \dots \ \mathbf{s}_{D-1}^{\mathrm{T}}]^{\mathrm{T}}, \tag{11}$$

where $\boldsymbol{\Psi}_d \in \mathbb{C}^{L \times L_D}$ is the $d$th precoding matrix and $\mathbf{s}_d$ is the $d$th $Q$-sparse signal of length $L_D$. In addition, for normalization purposes, we assume that the norm of each column of $\boldsymbol{\Psi}_d$ is 1.

To achieve a diversity gain, we assume that each $\mathbf{s}_d$ has the same support as

$$\mathcal{I} = \mathrm{supp}(\mathbf{s}_d), \ d = 0, \dots, D-1. \tag{12}$$

On the other hand, we assume that the values of the non-zero elements of each $\mathbf{s}_d$ are different from each other and convey independent data symbols as conventional IM. In this case, the number of bits per signal block $\mathbf{s}$ is given by

$$N_{\mathrm{b}}(D) = \lfloor \log_2 \binom{L_D}{Q} \rfloor + DQ \log_2 M. \tag{13}$$

In general, $N_{\mathrm{b}}(D)$ increases with $D$ for given $L_D$ and $Q$. In particular, for sparse IM with fixed small $Q$, more bits can be transmitted with a larger $D$, while the low-complexity CS detector can be used. Thus, in this section, we mainly consider the case where $Q$ is fixed and small, i.e., sparse IM.

We assume that $K = \frac{L}{D}$ is an integer (i.e., $L$ is a multiple of $K$) and consider $D$ disjoint index sets as follows:

$$\bigcup_{d=0}^{D-1} \mathcal{A}_d = \{0, \dots, L-1\} \text{ and } \mathcal{A}_d \cap \mathcal{A}_{d'} = \emptyset \tag{14}$$

with $|\mathcal{A}_d| = K$. For example, $\mathcal{A}_0 = \{0, 1 \dots, K-1\}$, $\mathcal{A}_1 = \{K, K+1 \dots, 2K-1\}$, and so on. Let $\mathbf{E}_d^{\mathrm{H}}$ be the submatrix of $\mathbf{F}$ taking the row vectors whose indices in $\mathcal{A}_d$. Thus, $\mathbf{E}_d = [\mathbf{F}^{\mathrm{H}}]_{\mathcal{A}_d}$ and $\mathbf{E}_d^{\mathrm{H}} \in \mathbb{C}^{K \times L}$. In addition, let

$$\mathbf{H}_d = \mathrm{diag}(H_{\mathcal{A}_d(0)}, \dots, H_{\mathcal{A}_d(K-1)}), \tag{15}$$

where $\mathcal{A}_d(k)$ denotes the $k$th element of $\mathcal{A}_d$. Then, we can show that

$$\begin{aligned} \mathbf{y}_d &= [\mathbf{y}]_{\mathcal{A}_d} \\ &= [\mathbf{HFs} + \tilde{\mathbf{n}}]_{\mathcal{A}_d} \\ &= \mathbf{H}_d \mathbf{E}_d^{\mathrm{H}} \mathbf{s} + \tilde{\mathbf{n}}_d, \ d = 0, \dots, D-1, \end{aligned} \tag{16}$$

where $\tilde{\mathbf{n}}_d = [\tilde{\mathbf{n}}]_{\mathcal{A}_d}$. Substituting (11) into (16), we have

$$\mathbf{y}_d = \mathbf{H}_d \mathbf{E}_d^{\mathrm{H}} \boldsymbol{\Psi}_d \mathbf{s}_d + \sum_{q \neq d} \mathbf{H}_d \mathbf{E}_d^{\mathrm{H}} \boldsymbol{\Psi}_q \mathbf{s}_q + \tilde{\mathbf{n}}_d. \tag{17}$$

We now consider a special case that the precoding matrices satisfy the following condition:

$$\mathbf{E}_d^{\mathrm{H}} \boldsymbol{\Psi}_q = \mathbf{0}, \ \forall d \neq q. \tag{18}$$

From (18), we can have

$$\mathbf{y}_d = \mathbf{H}_d \mathbf{E}_d^{\mathrm{H}} \boldsymbol{\Psi}_d \mathbf{s}_d + \tilde{\mathbf{n}}_d. \tag{19}$$

It is not difficult to find the precoding matrices that satisfy (18). For example, if $\boldsymbol{\Psi}_q = \mathbf{E}_q$ with $L_D = \frac{L}{D}$, we have

$$\mathbf{E}_d^{\mathrm{H}} \boldsymbol{\Psi}_q = \delta_{d,q} \mathbf{I},$$

where $\delta_{d,q}$ represents the Kronecker delta, and $\mathbf{y}_q = \mathbf{H}_q \mathbf{s}_q + \tilde{\mathbf{n}}_q$. Since this corresponds to the MCIM in [1] if $D = 1$, the resulting approach with $D \geq 1$ and $L_D = \frac{L}{D}$ can be seen as a generalization of MCIM with diversity gain, which is referred to as diversity MCIM (DMCIM) for convenience.

In general, if

$$\mathrm{Range}(\boldsymbol{\Psi}_d) \subseteq \mathrm{Range}(\mathbf{E}_d), \tag{20}$$

(18) holds. To obtain such precoding matrices, suppose that $\mathbf{C}_d$ is an $L \times L_D$ matrix of independent CSCG random variables. Let

$$\tilde{\boldsymbol{\Psi}}_d = \mathbf{P}_d \mathbf{C}_d \tag{21}$$

where $\mathbf{P}_d = \mathbf{E}_d (\mathbf{E}_d^{\mathrm{H}} \mathbf{E}_d)^{-1} \mathbf{E}_d^{\mathrm{H}} = \mathbf{E}_d \mathbf{E}_d^{\mathrm{H}}$ is the projection matrix that gives the vector space projection from the subspace $\mathrm{Range}(\mathbf{E}_d)$. Then, we can see that $\mathrm{Range}(\tilde{\boldsymbol{\Psi}}_d) \subseteq \mathrm{Range}(\mathbf{E}_d)$. To have $\boldsymbol{\Psi}_d$ from $\tilde{\boldsymbol{\Psi}}_d$, each column of $\tilde{\boldsymbol{\Psi}}_d$ can be normalized. The resulting IM is referred to as diversity random precoding IM (DRPIM) in this paper.

### B. A Low-Complexity MMV CS Detection Method

Since the support of $\mathbf{s}_d$ is the same for all $d$ in (19), the estimation of the support of $\mathbf{s}$ can be seen as a multiple measurement vector (MMV) problem [19] [20]. However, the measurement matrix is different for each $d$. In this case, unfortunately, the rank-aware MMV algorithms in [20] that provide better performance than OMP variations cannot be used for the signal detection. Thus, we may use a modified simultaneous OMP (SOMP) algorithm, where the original SOMP algorithm is proposed in [20], to recover the support of $\mathbf{s}_d$. For convenience, let $\boldsymbol{\Phi}_d = \mathbf{H}_d \mathbf{E}_d^{\mathrm{H}} \boldsymbol{\Psi}_d$. A pseudo-code for (modified) SOMP is summarized as follows.

**SOMP Algorithm**

*) Inputs: $\{\mathbf{y}_d\}$, $\{\boldsymbol{\Phi}_d\}$, and $Q$

0) Initialize: $\boldsymbol{\zeta}_d(0) = \mathbf{y}_d$, $\mathbf{e}_d(0) = \mathbf{0}$, and $\mathcal{T}_{(0)} = \emptyset$

1) for $i = 1 : Q$

2) $\quad \boldsymbol{\beta}_d(i) = \boldsymbol{\Phi}_d^{\mathrm{H}} \boldsymbol{\zeta}_d(i-1)$

3) $\quad p^* = \mathrm{argmax}_p \sum_d \left| [\boldsymbol{\beta}_d(i)]_p \right|^2$

4) $\quad \mathcal{T}_{(i)} = \mathcal{T}_{(i-1)} \cup p^*$

5) $\quad [\mathbf{e}_d(i)]_{\mathcal{T}} = [\boldsymbol{\Phi}_d]^{\dagger}_{\mathcal{T}_{(i)}} \mathbf{y}_d$

6) $\quad \boldsymbol{\zeta}_d(i) = \mathbf{y}_d - \boldsymbol{\Phi}_d \mathbf{e}_d(i)$

7) end;

8) Output: $\mathcal{T}_{(Q)}$

## V. Simulation Results

In this section, we present simulation results when 4-QAM (in this case, $M = 4$) is used for active signals. For the multipath channel, we assume that the channel coefficients are independent and $h_p \sim \mathcal{CN}(0, 1/P)$. For the CS detection, the SOMP algorithm is employed.

In Fig. 2, we assume that $L = 512$, $P = 10$, and $Q = 3$ and show the index error rate (IER) for different values of signal-to-noise ratio (SNR), which is given by

$$\frac{E_\mathrm{b}}{N_0} = \frac{Q\sigma_s^2}{N_\mathrm{b} N_0} = \frac{Q\gamma}{N_\mathrm{b}},$$

where $E_\mathrm{b}$ denotes the bit energy. Since MCIM cannot have the path diversity gain as OFDM, its performance is poor, which is also true although the ML detector is used as shown in [3]. On the other hand, SCIM can provide good performances with the two low-complexity detection methods, the MMSE detector in Subsection III-A and the CS detector. We also note that the performance of the CS detector is better than that of the MMSE detector.

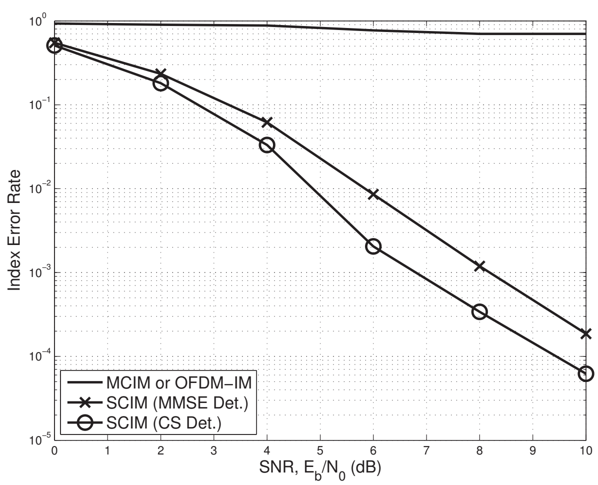



Fig. 2. IER for various values of SNR when $L = 512$, $P = 10$, and $Q = 3$. For the CS detector, we assume that $N = L$.

As mentioned in Subsection III-B, the complexity of the CS detector can be lower if a subset of the received signals is used. Fig. 3 shows the IER for different values of $\frac{N}{L}$ when $L = 512$, $P = 10$, $Q = 3$ and $\frac{E_\mathrm{b}}{N_0} = 6$ dB. We can see that the performance is degraded as $N$ decreases, while the complexity of the CS detector can be lower. Fortunately, however, the performance degradation is negligible when $\frac{N}{L} \geq 0.75$. Thus, in this case, about 25% complexity saving can be achieved without a significant performance degradation.

Fig. 4 shows IER for different numbers of multipaths, $P$, when $L = N = 128$, $Q = 3$, and $\frac{E_\mathrm{b}}{N_0} = 8$ dB. For DMCIM and DRPIM, we assume that $L_D = \frac{L}{D}$. DRPIM performs better than DMCIM as shown in Fig. 4. We also observe that DMCIM cannot exploit the path diversity gain[1] (as its

[1]However, as will be shown in Fig. 5, DMCIM can have the transmit diversity gain by the transmit diversity scheme.

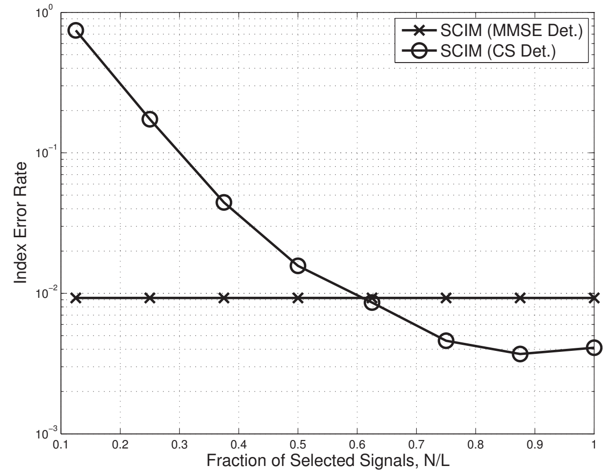



Fig. 3. IER for different ratios of the number of used signals to the total number of signals in the OMP detector when $L = 512$, $P = 10$, $Q = 3$ and $\frac{E_\mathrm{b}}{N_0} = 6$ dB. For the MMSE detector, we assume that all the received signals are used.

performance cannot be improved by the increase of $P$). On the other hand, SCIM and DRPIM have a better performance for a larger $P$. In addition, DRPIM outperforms SCIM due to the transmit diversity gain with more information bits (as $N_\mathrm{b} = 36$ and 24 in DRPIM and SCIM, respectively).

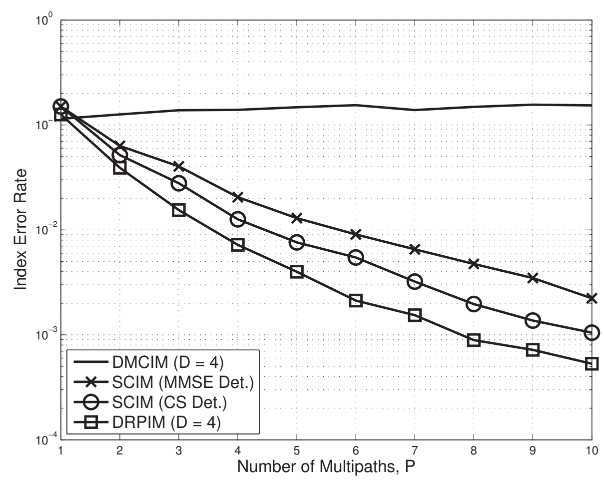
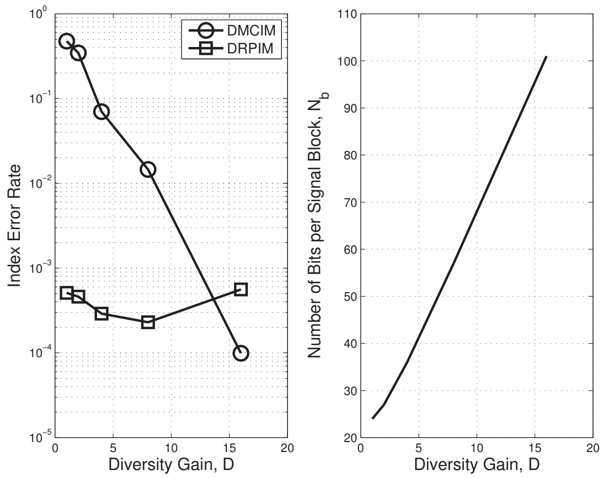



Fig. 4. IER for different numbers of multipaths, $P$, when $L = N = 128$, $Q = 3$ and $\frac{E_\mathrm{b}}{N_0} = 8$ dB.

The impact of $D$ on the performance of DMCIM and DRPIM is shown in Fig. 5 when $L = 128$, $P = 6$, $Q = 3$, $L_D = \frac{L}{D}$, and $\frac{E_\mathrm{b}}{N_0} = 10$ dB. As $D$ increases, the number of bits per signal block increases as shown in Fig. 5 (b). Fig. 5 (a) shows that DMCIM can transmit more bits per signal block and have a better performance as $D$ increases. Consequently, the transmit diversity scheme in Section IV can improve the performance of MCIM or OFDM-IM in terms of the number of information bits as well as IER. In DRPIM, we note that $D = 8$ can provide a best performance in terms of IER. It is also noteworthy that the transmit diversity scheme would be useful if the path diversity gain is limited (i.e., a small $P$) for DRPIM.




(a) (b)

Fig. 5. Performance of DMCIM and DRPIM for various values of $D$ when $L = 128$, $P = 6$, $Q = 3$, $L_D = \frac{L}{D}$, and $\frac{E_b}{N_0} = 10$ dB: (a) IER; (b) the number of information bits per signal block.

## VI. Concluding Remarks

In this paper, we proposed SCIM as a variation of OFDM-IM or MCIM. Since SCIM can inherit the advantages of SC systems over MC systems, it can perform better than MCIM by exploiting the path diversity gain under a multipath fading environment. For sparse signals, we have shown that CS algorithms (e.g., the OMP algorithm) can be employed for low-complexity signal detection. A transmit diversity scheme was proposed for both sparse MCIM and SCIM not only to improve the performance in terms of the diversity gain, but also to increase the number of information bits per signal block. From simulation results, we confirmed that the transmit diversity scheme is effective in improving the IER performance and increasing the number of bits per signal block.

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