On the Stability and Throughput of Compressive Random Access in MTC

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***Abstract*—Compressive random access has been considered for machine-type communications (MTC) as it has a potential to support massive connectivity by exploiting the sparsity of device activity. However, its performance limitations are not well studied yet compared with other well-known candidates for MTC, e.g., multichannel ALOHA. In this paper, we investigate the stability of compressive random access with a controlled access probability strategy and derive the maximum stable throughput to see performance limitations. From the analysis results, we can show that the maximum stable throughput of compressive random access is higher than that of multichannel ALOHA by a factor of 2. As a result, we can claim that compressive random access can support more devices than multichannel ALOHA in MTC.**

***Index Terms*—machine-type communications; random access; stability; compressive sensing**

I. Introduction

To support a number of devices for connection to a data aggregator or access point (AP), there has been a growing interest in machine-type communications (MTC) or machine-to-machine (M2M) communications [1], [2]. MTC can be used for various applications from smart grid to health care and be implemented within cellular systems as in [3], where random access has been considered for MTC due to different characteristics of traffics in MTC from those in traditional human-type communications (HTC) (e.g., few short packets per machine and sporadic transmissions).

For MTC, various random access schemes are considered. Based on the stabilized multichannel ALOHA in [4], random access for MTC has been studied in [5]–[7]. As shown in [5], multichannel ALOHA has a scaling property that the maximum stable throughput increases linearly with the number of multiple channels. While most multichannel ALOHA schemes can be stabilized by adaptively deciding the access probability as in [5]–[7] based on the approaches in [8], [9], it is also possible to consider the adaptive determination of the number of multiple channels as proposed in [10].

In multichannel ALOHA for MTC, the number of multiple channels is usually limited and much smaller than the number of devices. Thus, in the contention-based random access channel (RACH) procedure in [3], a pool of preambles is used. A device with a packet randomly chooses a preamble in the preamble pool and transmits for connection to an access point (AP), where each preamble in the RACH procedure can be seen as a channel in multichannel ALOHA. If a preamble is selected by multiple devices, collision happens and no reliable recovery can be made at the AP. On the other hand, if only one device chooses a preamble, the AP can recover this signal well provided that all the preamble sequences are orthogonal or nearly orthogonal so that the multiple access interference (MAI) is negligible. Note that in this case, the complexity of the receiver at the AP is low as a bank of correlators could be used.

There are other approaches for random access in MTC based on the notion of compressive sensing (CS) [11], [12], which are studied under various settings as in [13]–[15]. Random access based on CS is often called compressive random access [15] and it differs from multichannel ALOHA as it does not require the (near) orthogonality of multiple channels. From this, compressive random access can have more (non-orthogonal) multiple channels than multichannel ALOHA for a given radio resource at the cost of increasing receiver's complexity to mitigate the MAI. In compressive random access, the AP needs to use CS based multiuser detection (MUD) that can not only detect the signals from active devices, but also identify them (through their signature sequences).

In this paper, we study the stability of compressive random access based on the analysis approaches in [5]–[7]. To the best of our knowledge, the maximum stable throughput of compressive random access is not studied yet, while compressive random access has been extensively studied and applied to MTC [13]–[17]. Through the stability analysis, we can show that the maximum stable throughput of compressive random access is higher than that of multichannel ALOHA by a factor of 2, which makes compressive random access more suitable for MTC than conventional multichannel ALOHA.

Notation: Matrices and vectors are denoted by upper- and lower-case boldface letters, respectively. The superscripts T and H denote the transpose and complex conjugate, respectively. The p-norm of a vector $\mathbf{a}$ is denoted by $||\mathbf{a}||_p$ (If $p = 2$, the norm is denoted by $||\mathbf{a}||$ without the subscript). $\mathbb{E}[\cdot]$ and $\mathrm{Var}(\cdot)$ denote the statistical expectation and variance, respectively. $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ represents the distribution of circularly symmetric complex Gaussian (CSCG) random vectors with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{R}$.

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II. System Model

Suppose that a system consists of one AP and K devices. A device becomes active if it has a packet to send the AP. When the number of active devices at a time is small and each active device has a short packet to send to the AP, it is desirable to use random access as the signaling overhead is low. In this section, we present a compressive random access scheme based on [14]–[16] for narrowband signals, where the AP is able to carry out MUD without knowing active devices. Note that the system model for wideband signals can be found in [14] [17] [18]. For comparison purposes, we also briefly discuss multichannel ALOHA in this section.

A. Compressive Random Access

For compressive random access, suppose that each device has a unique signature or spreading code. Since the number of devices is large, spreading codes would not be orthogonal to each other. Denote by $\phi_k \in \mathbb{C}^L$ the spreading code of length L for device k. Each active device transmits spread signals of T data symbols. Throughout the paper, we assume that a packet consists of T data symbols. The tth received signal at the AP when active devices transmit their packets becomes

$$\mathbf{y}_t = \sum_{k \in \mathcal{I}} \phi_k h_k s_{k,t} + \mathbf{n}_t, \ t = 0, \ldots, T-1. \tag{1}$$

where $\mathcal{I}$ represents the index set of active devices, h_k is the channel coefficient from device k to the AP, $s_{k,t}$ is the tth data symbol from device k, and $\mathbf{n}_t \sim \mathcal{CN}(0, N_0\mathbf{I})$ is the background noise. In (1), we have assumed flat fading with a coherence time that is longer than the duration of T data symbols. In MTC, the data rate from devices might be sufficiently low (i.e., narrowband signals) and the signals may experience flat fading. Furthermore, in general, since a device is immobile and has a short packet, the packet duration would be shorter than the coherence time.

Let

$$\mathbf{\Phi} = [\phi_1 \ \ldots \ \phi_K] \in \mathbb{C}^{L \times K}.$$

Suppose that there are M active devices, i.e., $M = |\mathcal{I}|$. In addition, let $x_{k,t} = h_k s_{k,t}$ for $k \in \mathcal{I}$ and $x_{k,t} = 0$ for $k \notin \mathcal{I}$. Then, we have

$$\mathbf{y}_t = \mathbf{\Phi}\mathbf{x}_t + \mathbf{n}_t, \ t = 0, \ldots, T-1, \tag{2}$$

where $\mathbf{x}_t = [x_{1,t} \ \ldots \ x_{K,t}]^{\mathrm{T}}$ is M-sparse. In the context of CS, $\mathbf{\Phi}$ is referred to as the measurement matrix [19].

B. Multichannel ALOHA

In compressive random access, the ϕ_k's are not assumed to be orthogonal. Thus, when $L < K$, the AP is to exploit the sparsity of $\mathbf{x}_t$ to recover the signals transmitted by active devices. On the other hand, in conventional multichannel ALOHA, it is assumed that there are L orthogonal (or nearly orthogonal) preambles (or spreading codes) in a preamble pool to form L multiple channels that are available to all devices. An active device is to randomly choose one preamble from the preamble pool (or one channel out of L channels) for access [3]. Let $\mathbf{c}_l$ denote the lth preamble in the preamble pool and $k(q)$ denote the index of the qth active device. In addition, denote by $l(q)$ the index of preamble chosen by the qth active device. Then, in multichannel ALOHA, the received signal at the AP becomes

$$\mathbf{z} = \sum_{q=1}^{M} \mathbf{c}_{l(q)} h_{k(q)} s_{k(q)} + \mathbf{n}. \tag{3}$$

If $\mathbf{c}_l$'s are orthogonal, the AP can recover multiple signals using a bank of correlators. Let $\mathbf{C} = [\mathbf{c}_1 \ \ldots \ \mathbf{c}_L]$ and assume that $\mathbf{c}_l$ is orthonormal. Then, the outputs of the bank of L correlators with the input $\mathbf{z}$ in (3) become

$$\begin{aligned} \mathbf{r} &= [r_1 \ \ldots \ r_L]^{\mathrm{T}} \\ &= \mathbf{C}^{\mathrm{H}}\mathbf{z}, \end{aligned} \tag{4}$$

where $r_l = \mathbf{c}_l^{\mathrm{H}}\mathbf{z}$. It can be readily shown that

$$r_l = \sum_{k \in \mathcal{I}_l} h_k s_k + \mathbf{c}_l^{\mathrm{H}}\mathbf{n}, \tag{5}$$

where $\mathcal{I}_l$ is the index set of the active devices that choose the lth preamble. From each output of the bank of correlators (which is MAI-free as $\mathbf{c}_l$'s are orthogonal), the AP can see what preambles are chosen. If a preamble is chosen by a device, there is no collision and this device can be successfully connected. On the other hand, if a preamble is chosen by multiple devices, there is collision and those devices are backlogged. In this multichannel ALOHA, there is no need to exploit the sparsity of active devices in detecting signals and the receiver's complexity is low as a bank of correlators is used.

For simplicity, throughout the paper, we assume that the receiver at the AP in multichannel ALOHA can perfectly work to detect whether or not there is signal in r_l. In addition, it can also perfectly detect collision if there are multiple signals in r_l. For this simplicity, we may assume that $P = |h_k s_k|^2$, $k \in \mathcal{I}$, is sufficiently larger than $\mathrm{Var}(\mathbf{c}_l^{\mathrm{H}}\mathbf{n})$ (i.e., a high signal-to-noise ratio (SNR) is assumed). This assumption (of high SNR) is adopted in this paper to mainly compare the throughputs of multichannel ALOHA and compressive random access (not to see the performance of compressive random access itself). Note that this assumption is widely used to find the throughput of multichannel ALOHA as in [4], [5], [7].

III. CS based MUD

In compressive random access, since $K > L$ and the set of active devices, $\mathcal{I}$, is not known in advance, CS based MUD needs to be used at the AP to exploit the sparsity of active devices for multiple signal detection as well as active device identification. In this section, we study the MUD problem as a sparse multiple measurement vector (MMV) problem [20], [21].

Suppose that the AP is to detect signals, once all the packets from active devices are received. Let $\mathbf{Y} = [\mathbf{y}_0 \ \ldots \ \mathbf{y}_{T-1}]$, $\mathbf{X} = [\mathbf{x}_0 \ \ldots \ \mathbf{x}_{T-1}]$, and $\mathbf{N} = [\mathbf{n}_0 \ \ldots \ \mathbf{n}_{T-1}]$. Then, (2) becomes

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{X} + \mathbf{N}. \tag{6}$$

The estimation of $\mathcal{I}$ for the active device identification from (6) is a typical MMV problem as the support of $\mathbf{x}_t$ is the same for all t. We now assume that the SNR is sufficiently high. In this case, a sufficient and necessary condition to estimate $\mathcal{I}$ [20], [21] is given by

$$M < \frac{\text{spark}(\mathbf{\Phi}) - 1 + \text{rank}(\mathbf{X})}{2}, \tag{7}$$

where $\text{spark}(\mathbf{\Phi})$ is the smallest number of columns from $\mathbf{\Phi}$ that linearly dependent [22]. It can be readily shown that

$$\text{rank}(\mathbf{X}) \le \min\{M, T\}.$$

In general, since $s_{k,t}$ is iid, we have $\text{rank}(\mathbf{X}) = M$ if $T \ge M$ with probability (w.p.) 1. From this, (7) is reduced to

$$M \le \tau \stackrel{\triangle}{=} \text{spark}(\mathbf{\Phi}) - 2, \tag{8}$$

where τ is the sparsity threshold for a recovery guarantee. If $\mathbf{\Phi}$ is random, $\text{spark}(\mathbf{\Phi}) - 1 = \text{rank}(\mathbf{\Phi}) = L$ w.p. 1. In this case, we have

$$\tau = L - 1. \tag{9}$$

On the other hand, if $\mathbf{\Phi}$ is deterministic, we have [22] $\text{spark}(\mathbf{\Phi}) \ge 1 + \frac{1}{\mu(\mathbf{\Phi})}$, where $\mu(\mathbf{\Phi})$ is the mutual coherence of $\mathbf{\Phi}$. Thus, in this case, a lower-bound on τ is given by $\tau \ge \frac{1}{\mu(\mathbf{\Phi})}$. For Zadoff-Chu (ZC) sequences, there can be up to $K = L(L-1)$ column vectors for $\mathbf{\Phi}$ with mutual coherence $\frac{1}{\sqrt{L}}$ [23], where $L \ge 3$ is a prime. For Alltop sequences, the same mutual coherence can be obtained with up to $K = L^2$ column vectors for $\mathbf{\Phi}$ [24], [25].

IV. Stability and Throughput Analysis

In this section, we study the stability and throughput of compressive random access and compare them with those of multichannel ALOHA. In Sections II and III, we only consider one slot. However, in order to study the stability, we need to consider the variation of the total number of packets in the system over the time. Thus, we assume that the transmission of a packet from an active device is carried out within one slot and use j to denote the index of time slot.

A. Multichannel ALOHA

For multichannel ALOHA in this subsection, we consider the model in [4], [5], [7] and the following assumptions.

- **A1)** The number of new total packets at the end of slot j or total arrival rate, denoted by $A(j)$, is assumed to be iid with $\mathbb{E}[A(j)] = \nu$ and $\mathbb{E}[A^2(j)] < \infty$ (we also assume that a device that has a backlogged packet cannot have a new one).
- **A2)** The number of orthogonal channels is L in multichannel ALOHA. If there are multiple packets transmitted in a channel, packet collision happens and the AP cannot detect them all.
- **A3)** The estimated number of active devices during slot j, denoted by $S(j)$, is available at all the devices.

Denote by $N(j)$ the number of devices with packets to send at slot j. In general, the estimation of $N(j)$ for $S(j)$ plays a crucial role in stabilizing ALOHA [8], [26] or multichannel ALOHA [4]–[7]. In most approaches, each device is to obtain $S(j)$ from a ternary-feedback (0 for idle, 1 for successful recovery, and e for collision, which indicates there are more than 1 active device) per channel from the AP using a recursion formula. In [10], a different approach to find $S(j)$ at the AP is studied using the maximum likelihood (ML) formulation. Once $S(j)$ is obtained at the AP, it can be fed back to all devices. However, for simplicity, we do not further discuss approaches to obtain $S(j)$ in this paper, while $S(j)$ is assumed to be a reliable estimate of $N(j)$ and be available at the devices as stated in **A3)**.

Under **A1)**, $N(j)$ becomes a Markov chain with

$$N(j+1) = N(j) - D(j) + A(j), \tag{10}$$

where $D(j)$ represents the number of successfully recovered packets at the AP. To stabilize multichannel ALOHA, in [4], [5], [7], the access probability, which is the probability that a device with a packet transmits its packet, is to be adaptively decided as

$$p(j) = \min\{1, \frac{L}{S(j)}\}. \tag{11}$$

It is also shown in [5] that this access probability minimizes the number of backlogged devices under **A1)**.

Suppose that the controlled access probability in (11) is employed for multichannel ALOHA. Then, from [5], under **A1) – A3)**, the drift of $N(j)$ is given by

$$\begin{aligned} &\mathbb{E}[N(j+1) - N(j) \,|\, N(j) = n, S(j) = s] \\ &= \nu - \frac{nL}{s}\left(1 - \frac{1}{s}\right)^{n-1}. \end{aligned} \tag{12}$$

As $n \to \infty$ with fixed $z = \frac{n}{s}$, we have

$$\begin{aligned} &\mathbb{E}[N(j+1) - N(j) \,|\, N(j) = n, S(j) = s] \\ &\to \tilde{N}_{\text{ma}}(n, s) = \nu - Lze^{-z}, \end{aligned} \tag{13}$$

where $\tilde{N}_{\text{ma}}$ represents the asymptotic drift of multichannel ALOHA. Provided that $n = s$ (or $z = 1$, which minimizes the drift), we have

$$\tilde{N}_{\text{ma}}(n, s) = \nu - Le^{-1}.$$

Since the multichannel ALOHA becomes stable if $\tilde{N}_{\text{ma}}(n, s) < 0$, when the throughput function in terms of z is given by $T_{\text{ma}}(z) = Lze^{-z}$, the maximum stable throughput becomes

$$\hat{\nu}_{\text{ma}} = \max_z T_{\text{ma}}(z) = Le^{-1}, \tag{14}$$

which shows that the maximum stable throughput grows linearly with the number of multiple channels, L.

B. Compressive Random Access

In this subsection, we focus on the stability analysis of the compressive random access scheme when CS based MUD is used to recover multiple transmitted packets under the following assumption.

A2′) The AP is able to detect all the packets from active devices when the condition in (7) holds.

Under **A1)** and **A3)**, in compressive random access, we have

$$N(j+1) = N(j) - U(j) + A(j), \quad (15)$$

where $U(j)$ is the number of the packets that are successfully recovered at the AP. For tractable analysis, under **A2′)**, U is given by

$$U = M\mathbb{1}(M \leq \tau), \quad (16)$$

where $\mathbb{1}(\cdot)$ is the indicator function. That is, if the number of active devices is less than or equal to the sparsity threshold τ in (8), the AP can recover them successfully, while if M is greater than τ, the AP cannot recover them at all. This might be a bit pessimistic as some packets can be recovered although $M > \tau$ in practice. However, this can simplify the analysis.

The drift of $N(j)$ in compressive random access can be found as

$$\begin{aligned} \tilde{N}_{\mathrm{cra}}(n,s) &= \mathbb{E}[N(j+1) - N(j) \,|\, N(j) = n, S(j) = s] \\ &= \nu - \bar{U}(n,s), \end{aligned} \quad (17)$$

where $\bar{U}(n,s) = \mathbb{E}[U(j) \,|\, N(j) = n, S(j) = s]$. In compressive random access, we assume that the access probability is given by

$$p(j) = \min\{1, \frac{L_{\mathrm{eff}}}{S(j)}\}, \quad (18)$$

where L_{eff} represents the effect number of multiple channels, which is a design parameter. Note that (18) is similar to (11) except that the number of multiple channels, L, is replaced with L_{eff}.

For a sufficiently large s, we have

$$\begin{aligned} \bar{U}(n,s) &= \sum_{m=0}^{\tau} m \binom{n}{m} p^m (1-p)^{n-m} \\ &= np \sum_{m=1}^{\tau} \binom{n-1}{m-1} p^{m-1} (1-p)^{n-m} \\ &= np \sum_{m=0}^{\tau-1} \binom{n-1}{m} p^{m} (1-p)^{(n-1)-m}, \end{aligned} \quad (19)$$

where $p = \frac{L_{\mathrm{eff}}}{s}$. Suppose that $n \to \infty$ with fixed n/s. Let $\kappa = np$. Then, using the Poisson approximation for a large n [27], (19) can be approximated by $\bar{U}(n,s) = \kappa \Pr(X_\kappa \leq \tau - 1)$, where X_κ represents the Poisson random variable with parameter κ. Thus, the drift of compressive random access becomes $\tilde{N}_{\mathrm{cra}}(n,s) = \nu - \kappa \sum_{m=0}^{\tau-1} \frac{e^{-\kappa}\kappa^m}{m!}$. As a result, the maximum stable throughput with $\tilde{N}_{\mathrm{cra}}(n,s) < 0$ (for stable compressive random access) is given by

$$\hat{\nu}_{\mathrm{cra}} = \max_{\kappa \geq 0} \kappa e^{-\kappa} \sum_{m=0}^{\tau-1} \frac{\kappa^m}{m!}. \quad (20)$$

From (14) and (20), we can now compare the maximum stable throughputs of compressive random access and multichannel ALOHA. In Fig. 1, when $L = 50$, we show the throughput curves of multichannel ALOHA and compressive random access in terms of z and κ, respectively. In compressive random access, we assume that $\tau = L - 1$. It is shown that the maximum stable throughput of compressive random access can be higher than that of multichannel ALOHA.

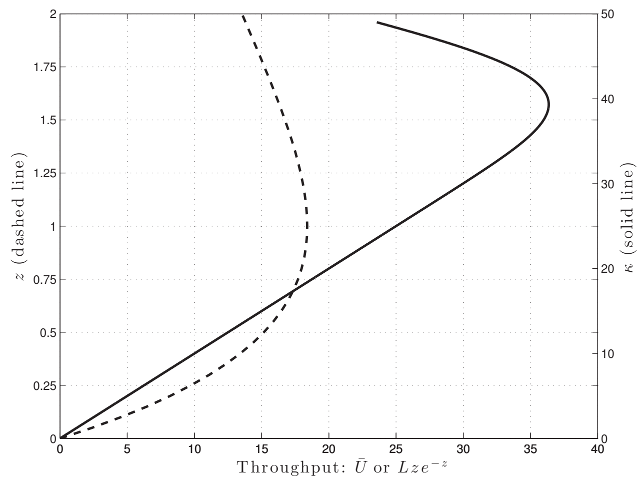

Fig. 1. Throughput curves of multichannel ALOHA and compressive random access (with $\tau = L - 1$) when $L = 50$.

Fig. 2 shows the maximum stable throughputs of multichannel ALOHA and compressive random access (with $\tau = L - 1$) for various values of L. In general, the maximum stable throughput grows linearly with L in both compressive random access and multichannel ALOHA. Furthermore, compressive random access has about two times higher maximum stable throughput than multichannel ALOHA. This clearly shows the advantage of compressive random access over multichannel ALOHA at the cost of increasing receiver's complexity.

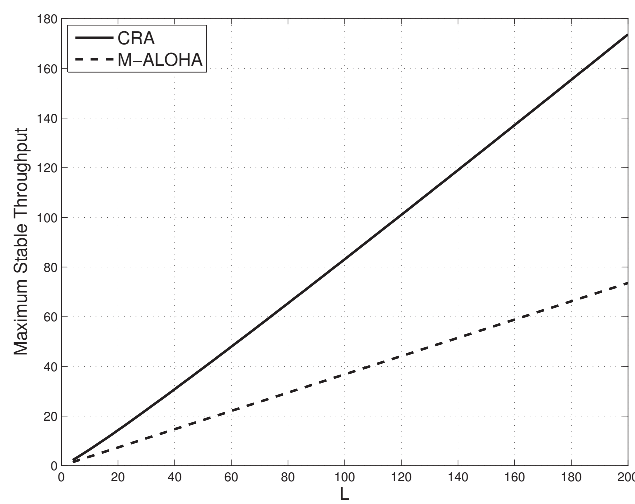

Fig. 2. Maximum stable throughputs of multichannel ALOHA and compressive random access (with $\tau = L - 1$) for various values of L.

Note that in Figs. 1 and 2 we assume that $\tau = L - 1$, which might be valid for random $\mathbf{\Phi}$.

In (18), for compressive random access, it is important to decide L_{eff}, which is a design parameter, to have the maximum throughput. If $n = s$, for given τ, the optimal effective number of multiple channels, denoted by $L^*_{\text{eff}}(\tau)$, which results in the maximum stable throughput, can be found as

$$L^*_{\text{eff}}(\tau) = \underset{\kappa \geq 0}{\operatorname{argmax}}\, \kappa e^{-\kappa} \sum_{m=0}^{\tau-1} \frac{\kappa^m}{m!}. \tag{21}$$

A numerical approach can be used to find $L^*_{\text{eff}}(\tau)$. In this case, the following results are useful.

Lemma 1. *The objective function in* (21) *has a* $\cap$*-shape in terms of* κ*. In addition, for* $\tau \geq 2$*, we have*

$$\tau + 1 > L^*_{\text{eff}}(\tau + 1) > L^*_{\text{eff}}(\tau) \tag{22}$$

$$L^*_{\text{eff}}(\tau) \geq \frac{1+\sqrt{5}}{2}. \tag{23}$$

Proof: For convenience, let

$$G_n(\kappa) = \kappa e^{-\kappa} \sum_{m=0}^{n} \frac{\kappa^m}{m!} = \sum_{m=0}^{n} W_m(\kappa),$$

where $W_m(\kappa) = e^{-\kappa} \frac{\kappa^{m+1}}{m!}$. It can be readily shown that $W_m(\kappa)$ has a global maximum at $\kappa = m + 1$ and a $\cap$-shape. Let $\hat{\kappa}_n = \operatorname{argmax}_{\kappa \geq 0} G_n(\kappa)$. Since $G_n(\kappa)$ has also a $\cap$-shape and

$$G_n(\kappa) = G_{n-1}(\kappa) + W_n(\kappa),$$

the maximum of $G_n(\kappa)$ should lie between the maximum of $G_{n-1}(\kappa)$ and the maximum of $W_n(\kappa)$. Thus, we have

$$\hat{\kappa}_{n-1} < \hat{\kappa}_n < n + 1. \tag{24}$$

Clearly, since $L^*_{\text{eff}}(\tau) = \hat{\kappa}_{\tau-1}$, it leads to (22).

When $n = 1$, after some manipulations, we can show that

$$\hat{\kappa}_1 = L^*_{\text{eff}}(2) = \frac{1+\sqrt{5}}{2}.$$

From this together with (22), we can claim (23). Note that it can also be readily shown that $\hat{\kappa}_0 = L^*_{\text{eff}}(1) = 1$. ■

From Lemma 1, we can see that (21) has a unique solution and the search interval is bounded between $L^*_{\text{eff}}(\tau - 1)$ and τ for a given τ.

V. Simulation Results

In this section, we present simulation results under a high SNR assumption, where the noise is negligible (only the MAI is taken into account). For MUD at the AP in compressive random access, the rank aware recursive matching pursuit (RA-ORMP) algorithm [21], which is a low-complexity algorithm, is used. In multichannel ALOHA, we assume L orthogonal channels. In compressive random access, the elements of $\mathbf{\Phi}$ are assumed to be independent CSCG random variables with zero mean and variance $1/L$.

Fig. 3 shows $\mathbb{E}[N(j)]$ for various values of the arrival rate, ν, when $L = 50$. According to Fig. 1, the maximum stable throughputs of multichannel ALOHA and compressive random access are $\hat{\nu}_{\text{ma}} = 18.39$ and $\hat{\nu}_{\text{cra}} = 39.32$, respectively. In multichannel ALOHA, $\mathbb{E}[N(j)]$ approaches ∞ as $\nu \to \hat{\nu}_{\text{ma}}$. On the other hand, in compressive random access, we can see that the actual maximum stable throughput is lower than $\hat{\nu}_{\text{cra}}$, which results from the suboptimal performance of the RA-ORMP algorithm. However, we can still see that compressive random access can accept a higher arrival rate and provide a (nearly two-times) higher maximum stable throughput than multichannel ALOHA.

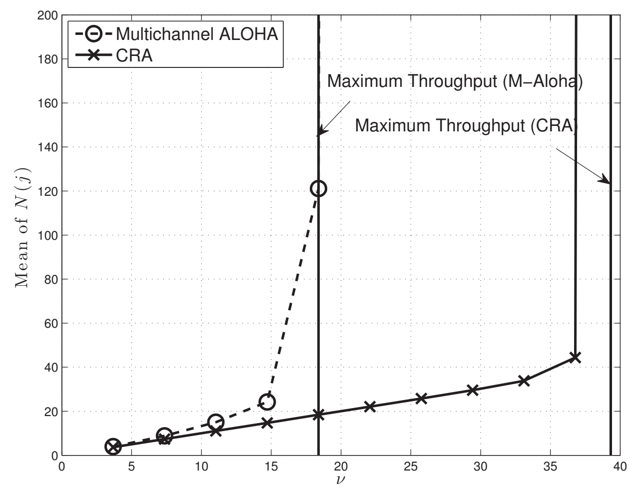

Fig. 3. $\mathbb{E}[N(j)]$ versus arrival rate for multichannel ALOHA and compressive random access (CRA) when $L = 50$.

Fig. 4 shows $\mathbb{E}[N(j)]$ for various values of L when $\nu = 50e^{-1}$. Since $\nu = 50e^{-1}$, the number of multiple channels, L, has to be greater than 50 to stabilize multichannel ALOHA, which can be confirmed by the curve of $\mathbb{E}[N(j)]$ of multichannel ALOHA in Fig. 4 (the dashed line). On the other hand, in compressive random access, a smaller L can also stabilize the system. This demonstrates that compressive random access can support a number of devices in MTC with a smaller number of multiple channels than multichannel ALOHA.

VI. Concluding Remarks

In this paper, we have studied the stability of compressive random access. As in stabilized multichannel ALOHA, the controlled access probability was considered to find the drift of the number of devices with packets. From the stability analysis, it was shown that the maximum stable throughput can be higher than that of multichannel ALOHA by a factor of two. Thus, with the same amount of radio resource for MTC, compressive random access could support more devices than multichannel ALOHA.

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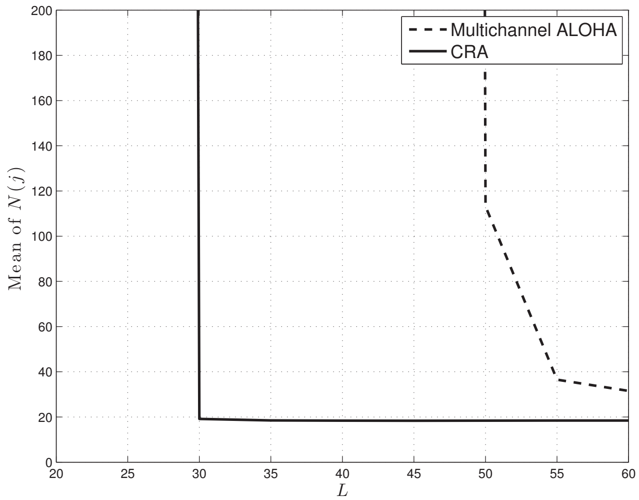

Fig. 4. $\mathbb{E}[N(j)]$ versus L for multichannel ALOHA and compressive random access (CRA) when $\nu = 50e^{-1}$.

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