Successive Hypothesis Testing Based Sparse Signal Recovery and Its Application to MUD in Random Access

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Abstract—**Based on successive hypothesis testing, we propose an approach for sparse signal recovery and apply it to random access to detect multiple block-sparse signals over frequency-selective fading channels. By introducing the sparsity variable, the proposed approach decides the presence or absence of the signal in each stage. To mitigate the error propagation, adaptive ordering is also employed as a greedy algorithm. From simulation results, it is shown that the proposed approach performs better than the block orthogonal matching pursuit algorithm, which is a well-known greedy compressive sensing algorithm for compressive random access.**

Index Terms—**Random access, sparse signal recovery.**

I. Introduction

FOR machine-type communications (MTC), random access has been considered in [1] and [2]. In random access, where the active devices cannot be known in advance, the notion of compressive sensing (CS) [3], [4] has been applied to multiuser detection (MUD) by exploiting the sparsity of active devices [5], [6]. The CS-based MUD approaches in [7]–[9] can not only detect the signals from active devices over frequency-selective fading channels, but also estimate their channel state information. As a result, the approaches in [7]–[9] could be well suited to MTC where devices of short packets want to transmit their signals over frequency-selective fading channels with low signaling overhead.

Since there are low-complexity approaches to recover sparse signals in CS such as greedy algorithms, we may readily develop low-complexity CS-based MUD approaches for random access. For example, the orthogonal matching pursuit (OMP) algorithm [10], which is a well-known greedy algorithm, can be used [7]. A key feature of the OMP algorithm is successive interference cancellation (SIC), which is also employed for multiple input multiple output (MIMO) detection [11], [12].

As shown in [7], MUD in random access over frequency-selective fading channels can be carried out via block-sparse signal recovery. For this MUD, we propose an approach based on successive hypothesis testing to decide the presence or absence of a signal in each stage using the sparsity variable. There are similar approaches based on Bayesian hypothesis testing for correlated signals [13]–[15], while no correlation of signals is considered in this letter. The main advantages of the proposed approach are that 1) it does not require the knowledge of the number of active signals; 2) it can take into account the prior information of the activity of signals. For a better performance by mitigating the error propagation, adaptive ordering is also employed, which results in a greedy algorithm similar to the OMP algorithm. The proposed approach is applied to random access and shown to have a better performance than a well-known CS greedy algorithm based on OMP from simulation results.

Notation: The superscripts T and H denote the transpose and complex conjugate, respectively. The ℓ-norm of a vector $\mathbf{a}$ is denoted by $||\mathbf{a}||_\ell$ (if $\ell = 2$, the norm is denoted by $||\mathbf{a}||$ without the subscript). $\mathbb{E}[\cdot]$ denotes the statistical expectation. $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ represents the distribution of circularly symmetric complex Gaussian (CSCG) random vectors with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{R}$.

II. CS-Based Detection in Random Access

In this letter, we mainly focus on MUD in random access over frequency-selective fading channels. Suppose that there are multiple devices for random access in a single-carrier system with cyclic-prefix (CP). As in [7], for random access, we assume that each device has a unique signature sequence. The set of signature sequences is given by $\mathcal{C} = \{\mathbf{c}_1, \dots, \mathbf{c}_L\}$, where $\mathbf{c}_l \in \mathbb{C}^{N\times 1}$ denotes the lth signature sequence, which is assigned to the lth device. We assume that only few devices become active at a time, while all the devices are synchronized. Denote by $l(m)$ the index of the mth active device and by M $(\le L)$ the number of active devices. The qth received signal at the access point (AP) after removing CP becomes

$$y_q = \sum_{m=1}^{M} \sum_{p=0}^{P-1} h_{l(m),p} c_{l(m),(q-p)_N} + n_q, \; q = 0, \dots, N-1 \tag{1}$$

where $h_{l,p}$ is the channel impulse response (CIR) from device l to the AP, P is the length of CIR, and $n_q \sim \mathcal{CN}(0, N_0)$ is the background noise. Here, $\mathbf{c}_l = [c_{l,0} \ \dots \ c_{l,N-1}]^{\mathrm{T}}$ represents the lth sequence in $\mathcal{C}$ and $(x)_N = x \pmod N$. Let $\mathbf{y} = [y_0 \ \dots \ y_{N-1}]^{\mathrm{T}}$, $\mathbf{n} = [n_0 \ \dots \ n_{N-1}]^{\mathrm{T}}$, and denote by $\mathbf{C}_l$

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the $N \times N$ circulant matrix of $\mathbf{c}_l$. From (1), we have

$$\mathbf{y} = \sum_{m=1}^{M} \mathbf{C}_{l(m);P}\mathbf{h}_{l(m)} + \mathbf{n} \tag{2}$$

where $\mathbf{C}_{l;P}$ represents the submatrix of $\mathbf{C}_l$ obtained by taking the first P columns, $\mathbf{h}_l = [h_{l,0} \ \ldots \ h_{l,P-1}]^{\mathrm{T}}$, and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ is the background noise vector. Let $\mathbf{\Psi}_l = \mathbf{C}_{l;P}$ and $\mathbf{\Psi} = [\mathbf{\Psi}_1 \ \ldots \ \mathbf{\Psi}_L] \in \mathbb{C}^{N\times PL}$. Then, (2) becomes

$$\mathbf{y} = \mathbf{\Psi}\mathbf{s} + \mathbf{n} \tag{3}$$

where the lth $P \times 1$ submatrix of $\mathbf{s}$ is given by $\mathbf{s}_l$, which is found as

$$\mathbf{s}_l = \begin{cases} \mathbf{h}_{l(m)}, & l \in \{l(1), \ldots, l(M)\}; \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

The sparse signal $\mathbf{s}$ is a concatenation of L subvectors of size $P \times 1$ as $\mathbf{s} = [\mathbf{s}_1^{\mathrm{T}} \ \mathbf{s}_2^{\mathrm{T}} \ \cdots \ \mathbf{s}_L^{\mathrm{T}}]^{\mathrm{T}}$ and the (block) sparsity [16] is given by $||\mathbf{s}||_{2,0} = \sum_{l=1}^{L} \mathbb{1}(||\mathbf{s}_l||_2 > 0)$, where $\mathbb{1}(\cdot)$ is the indicator function. Throughout the letter, we consider the block-sparse signal model in (3). Although it has been derived for random access over frequency-selective fading channels, it can also be a generic model for block-sparse signal recovery in [16].

In conventional MUD [17], $\mathbf{s}$ in (3) can be estimated under the assumption that all the signals are present (or all the devices are active). However, in random access, only few elements of $\mathbf{s}$ are nonzero. To take into account the sparsity of active devices, in [5] [6], low-complexity CS algorithms are considered for MUD from $\mathbf{y}$ when $P = 1$ (i.e., flat fading channels). For the case of $P \geq 1$, in [7] and [8], it is noted that the channel estimation has to be considered in conjunction with block-sparse signal recovery. Thus, in [7], the block-OMP (BOMP) algorithm, which is proposed in [16], is applied to MUD so that the AP can not only detect the active devices, but also estimate the nonzero channel vectors of the active devices.

It is important to note that the approaches in [7] and [8] for MUD in random access differ from conventional MTC random access such as the random access channel procedure [1], which is used to establish connections, not to transmit data sequences. The approaches in [7] and [8], which are often called compressive random access schemes, are one-shot transmission schemes that aim to transmit short packets without any handshaking procedure.

In compressive random access, under a frequency-selective fading environment (i.e., $P > 1$), we can consider the block-coherence of $\mathbf{\Psi}$ [16] when a CS algorithm is used to recover $\mathbf{s}$. The block-coherence of $\mathbf{\Psi}$ (where the norm of any column is normalized to be unity) is defined as [16]

$$\mu_{\mathrm{B}}(\mathbf{\Psi}) = \frac{1}{P} \max_{l \neq m} \rho(\mathbf{\Psi}_l^{\mathrm{H}}\mathbf{\Psi}_m)$$

where $\rho(\mathbf{X}) = \lambda_{\max}^{1/2}(\mathbf{X}^{\mathrm{H}}\mathbf{X})$. Here, $\lambda_{\max}(\mathbf{B})$ denotes the largest eigenvalue of the positive-semidefinite matrix $\mathbf{B}$. In general, the performance of CS algorithms can be better as the block-coherence decreases. Thus, noting that the pth column of $\mathbf{\Psi}_l = \mathbf{C}_{l;P}$ can be obtained by circular shifts of the first column or $\mathbf{c}_l$, we may use Zadoff–Chu [18] or Alltop sequences for $\mathcal{C}$ so that $\mathbf{\Psi}$ has a small block-coherence. In particular, with a prime number $N \geq 5$, if $\mathbf{c}_l$ s are orthogonal Alltop sequences [19], i.e.,

$$[\mathbf{c}_l]_n = \frac{1}{\sqrt{N}} e^{\frac{j2\pi}{N}(n^3+(l-1)n)}, \ n = 0, \ldots, N-1; \ l = 1, \ldots, L, \tag{4}$$

the block-coherence $\mathbf{\Psi}$ becomes $\rho_{\mathrm{B}}(\mathbf{\Psi}) = \frac{1}{\sqrt{N}}$.

Note that the block-sparsity in (3) differs from that in [13] (which is the first work that uses Bayesian hypothesis testing for sparse recovery) [14], [15] where the lengths of $\mathbf{s}_l$ s are random and correlated, while it is the same as that in [16] where the length of $\mathbf{s}_l$ is fixed and known. The approaches in [14] and [15] assume a Markov chain for the activity of the elements of $\mathbf{s}$, and employ Bayesian hypothesis testing for sparse recovery. Thus, those approaches are attractive when the structure of block-sparse signals is not known. On the other hand, in our model, the structure of block-sparse signals is known as mentioned earlier. In addition, since the activity of device is independent, the Bayesian hypothesis testing approaches in [14] and [15] are not suitable for the MUD in compressive random in this letter. In some applications, however, the activity of devices can be correlated (in some wireless sensor networks, where sensors are to detect spatial information), the correlated source model in [20], [14], and [15] could be considered for compressive random access, which is beyond the scope of this letter.

III. Successive Sparse-Signal Detection Approach With a Predetermined Order

As mentioned earlier, the BOMP algorithm can be considered to recover the block-sparse signal $\mathbf{s}$ with known[1] block-sparsity. However, in this section, we propose a different approach to recover the block-sparse signal $\mathbf{s}$ that does not require the block-sparsity. This approach is based on a low-complexity approach studied for MIMO detection [12].

Let $\mathbf{a}_l$ be the unknown channel vector for the lth active device. Then, (3) becomes

$$\mathbf{y} = \sum_{l=1}^{L} \mathbf{\Psi}_l\mathbf{a}_l u_l + \mathbf{n} \tag{5}$$

where u_l is the sparsity variable and $u_l \in \{0, 1\}$. If the lth device is inactive, $u_l = 0$.

Throughout the letter, we consider the following assumptions.

A1) $\mathbf{a}_l$ and u_l are independent, and $\{\mathbf{a}_l\}$ are mutually independent zero-mean CSCG random vectors (i.e., Rayleigh multipath fading channels are assumed) and $\{u_l\}$ are also mutually independent.

A2) The covariance matrix of $\mathbf{a}_l$ is known at the AP. In addition, $\Pr(u_l = 1) = \mathbb{E}[u_l] = \bar{u}_l$ is known.

Due to the sparsity, we expect that $\Pr(u_l = 1) \ll \Pr(u_l = 0)$. Under A1) and A2), we propose an algorithm based on successive hypothesis testing to recover the sparse signal when the detection order is given in this section. For convenience, we detect the signal in the increasing order. In Section IV, we will consider adaptive order.

Let $\mathbf{v}_m = \sum_{l=m+1}^{L} \mathbf{\Psi}_l\mathbf{a}_l u_l + \mathbf{n}$. Under A1) and A2), we assume $\mathbf{v}_m \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_m)$, where

$$\mathbf{R}_m = \mathbb{E}\left[\mathbf{v}_m\mathbf{v}_m^{\mathrm{H}}\right] = \sum_{l=m+1}^{L} \mathbf{\Psi}_l\mathbf{G}_l\mathbf{\Psi}_l\bar{u}_l + N_0\mathbf{I}. \tag{6}$$

[1] It is also possible to apply the BOMP algorithm without knowing the block-sparsity using some (*ad hoc*) termination conditions.

Here, $\mathbf{G}_l = \mathbb{E}[\mathbf{a}_l \mathbf{a}_l^{\mathrm{H}}]$. When $m = 1$, we consider the hypothesis testing for $u_m = u_1$ with the following likelihood function:

$$f(\mathbf{y} \,|\, u_1) = \begin{cases} C_0 e^{-(\mathbf{y}-\boldsymbol{\Psi}_1\mathbf{v}_1)^{\mathrm{H}}\mathbf{R}_1^{-1}(\mathbf{y}-\boldsymbol{\Psi}_1\mathbf{v}_1)}, & \text{if } u_1 = 1; \\ C_0 e^{-\mathbf{y}^{\mathrm{H}}\mathbf{R}_1^{-1}\mathbf{y}}, & \text{if } u_1 = 0 \end{cases} \tag{7}$$

where C_0 is the normalization constant. Then, the maximum *a posteriori* probability decision on u_1 becomes

$$\hat{u}_1 = \begin{cases} 1 & \text{if } \Pr(u_1 = 1 \,|\, \mathbf{y}) > \Pr(u_1 = 0 \,|\, \mathbf{y}); \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where $\Pr(u_1 \,|\, \mathbf{y}) = C_1 f(\mathbf{y} \,|\, u_1) \Pr(u_1)$. Here, C_1 is constant. Since $\mathbf{v}_1$ is not available, we can replace $f(\mathbf{y} \,|\, u_1 = 1)$ with $\max_{\mathbf{v}_1} f(\mathbf{y} \,|\, u_1 = 1, \mathbf{v}_1) = f(\mathbf{y} \,|\, u_1 = 1, \hat{\mathbf{v}}_1)$, where

$$\hat{\mathbf{v}}_1 = (\boldsymbol{\Psi}_1^{\mathrm{H}}\mathbf{R}_1^{-1}\boldsymbol{\Psi}_1)^{-1}\boldsymbol{\Psi}_1^{\mathrm{H}}\mathbf{R}_1^{-1}\mathbf{y}$$

is the maximum likelihood (ML) estimate of $\mathbf{v}_1$.

Once $\hat{u}_1$ and $\hat{\mathbf{v}}_1$ are found, the signal from the first device can be subtracted as $\mathbf{y}_{(1)} = \mathbf{y}_{(0)} - \boldsymbol{\Psi}_1\hat{\mathbf{v}}_1\hat{u}_1$, where $\mathbf{y}_{(0)} = \mathbf{y}$. In general, we have

$$\mathbf{y}_{(m)} = \mathbf{y}_{(m-1)} - \boldsymbol{\Psi}_m\hat{\mathbf{v}}_m\hat{u}_m, \tag{9}$$

which is the SIC that is used for MIMO detection except for the sparsity variable. Then, from $\mathbf{y}_{(m-1)}$, $\hat{u}_m$ can be obtained as

$$\hat{u}_m = \begin{cases} 1 & \text{if } \lambda_m > \tau_m; \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $\lambda_m = \ln \frac{f(\mathbf{y}_{(m-1)} \,|\, u_m = 1, \hat{\mathbf{v}}_m)}{f(\mathbf{y}_{(m-1)} \,|\, u_m = 0)}$, $\tau_m = \ln \frac{\Pr(u_m = 0)}{\Pr(u_m = 1)}$, and

$$\hat{\mathbf{v}}_m = (\boldsymbol{\Psi}_m^{\mathrm{H}}\mathbf{R}_m^{-1}\boldsymbol{\Psi}_m)^{-1}\boldsymbol{\Psi}_m^{\mathrm{H}}\mathbf{R}_m^{-1}\mathbf{y}_{(m-1)}.$$

The proposed approach to detect sparse signals based on successive hypothesis testing is referred to as the successive sparse signal detection (SSSD) approach in this letter. While this approach is similar to an approach in [11] and [12], which is called V-BLAST, due to SIC, the main difference from V-BLAST is the decision on the presence or absence of the signal in each stage to take into account the sparsity of activity, which is in (10). Note that the SSSD approach also suffers from the error propagation due to SIC.

In summary, the SSSD approach performs successive hypothesis testing for the presence or absence of each block-signal (of length P). If the signal is detected, we perform the ML estimation of the block-signal and its ML estimate is subtracted from the received signal for the next hypothesis testing. There are some key features of the SSSD approach. It can have different *a prior* probability of the presence of the signal, $\Pr(u_l)$, and does not require to know the number of signals, or sparsity. Thus, it can be applied to the case where M is random (and even M is not sparse).

IV. Successive Sparse-Signal Detection Approach With an Adaptive Order

It is well known that V-BLAST can have a better performance if the detection order can be adaptively decided at the cost of increasing complexity to mitigate the error propagation [21]. In this section, we propose an ordered SSSD approach.

In order to derive the SSSD approach with adaptive order, let $\mathcal{L}(0) = \{1, \ldots, L\}$, $\mathbf{y}_{(0)} = \mathbf{y}$, and

$$\mathbf{R}_{(0)} = \sum_{l=1}^{L} \boldsymbol{\Psi}_l\mathbf{G}_l\boldsymbol{\Psi}_l^{\mathrm{H}}\bar{u}_l + N_0\mathbf{I}.$$

The index for the stage of the successive detection is denoted by t in a pair of round brackets, i.e., (t).

Let $t = 1$. In order to decide the presence or absence of the signal vector to be detected at the (t)th stage, we need to perform the hypothesis testing for all $l \in \mathcal{L}(t-1)$. To this end, we consider the following log-likelihood ratio (LLR) of the lth signal vector:

$$\lambda_{l,(t)} = \ln \frac{f(\mathbf{y}_{(t-1)} \,|\, u_l = 1, \hat{\mathbf{v}}_{l,(t-1)})}{f(\mathbf{y}_{(t-1)} \,|\, u_l = 0)}, \; l \in \mathcal{L}(t-1)$$

where

$$\begin{aligned} \hat{\mathbf{v}}_{l,(t-1)} &= \left(\boldsymbol{\Psi}_l^{\mathrm{H}}\mathbf{R}_{l,(t-1)}^{-1}\boldsymbol{\Psi}_l\right)^{-1}\boldsymbol{\Psi}_l^{\mathrm{H}}\mathbf{R}_{l,(t-1)}^{-1}\mathbf{y}_{(t-1)} \\ \mathbf{R}_{l,(t-1)} &= \mathbf{R}_{(t-1)} - \boldsymbol{\Psi}_l\mathbf{G}_l\boldsymbol{\Psi}_l^{\mathrm{H}}. \end{aligned} \tag{11}$$

Here, $\mathbf{R}_{(t)} = \sum_{l \in \mathcal{L}(t)} \boldsymbol{\Psi}_l\mathbf{G}_l\boldsymbol{\Psi}_l^{\mathrm{H}}\bar{u}_l + N_0\mathbf{I}$.

After some manipulations, we can show that

$$\lambda_{l,(t)} = \mathbf{y}_{(t-1)}^{\mathrm{H}}\mathbf{W}_{l,(t-1)}\mathbf{y}_{(t-1)}, \tag{12}$$

where

$$\mathbf{W}_{l,(t-1)} = \mathbf{R}_{l,(t-1)}^{-1}\boldsymbol{\Psi}_l(\boldsymbol{\Psi}_l^{\mathrm{H}}\mathbf{R}_{l,(t-1)}^{-1}\boldsymbol{\Psi}_l)^{-1}\boldsymbol{\Psi}_l^{\mathrm{H}}\mathbf{R}_{l,(t-1)}^{-1}.$$

The signal to be decided can be chosen as follows:

$$l^*(t) = \underset{l \in \mathcal{L}(t)}{\operatorname{argmax}} \, |\lambda_{l,(t)} - \tau_l| \tag{13}$$

which has the largest gap from τ_l. Then, the corresponding sparsity variable can be decided by

$$\hat{u}_{l^*_{(t)}} = \begin{cases} 1, & \text{if } \lambda_{l^*(t),(t)} > \tau_l; \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Once the presence of the $l^*(t)$th signal is decided, this signal can be subtracted and $\mathcal{L}(t)$ can be updated as follows:

$$\begin{aligned} \mathbf{y}_{(t)} &= \mathbf{y}_{(t-1)} - \boldsymbol{\Psi}_{l^*(t)}\hat{\mathbf{v}}_{l^*(t)}\hat{u}_{l^*(t)} \\ \mathcal{L}(t) &= \mathcal{L}(t-1) \setminus l^*(t). \end{aligned} \tag{15}$$

The resulting approach is referred to as the ordered SSSD approach, which is also a greedy algorithm as the BOMP algorithm. The main difference from the BOMP algorithm is the decision step with the sparsity variable in (14), where the decision of the absence of signal is also made. In the BOMP algorithm, the matching pursuit is carried out only for active signals (or devices). On the other hand, in the ordered SSSD approach the matching pursuit is carried out for both active and inactive signals using the LLR as in (13) in the ordered decision, which could result in a better performance.

The computational complexity of the SSSD approach can be low if some key matrices are obtained in advance. For example, $\mathbf{R}_{(t)}$ and $\mathbf{R}_{l,(t-1)}$ can be found in advance as they depend only on $\boldsymbol{\Psi}_l$ and $\mathbf{G}_l$. Thus, $\mathbf{W}_{l,(t-1)}$ can be computed in advance and stored. In this case, the complexity mainly depends on finding $\hat{\mathbf{v}}_{l,(t-1)}$ with precomputed $(\boldsymbol{\Psi}_l^{\mathrm{H}}\mathbf{R}_{l,(t-1)}^{-1}\boldsymbol{\Psi}_l)^{-1}\boldsymbol{\Psi}_l^{\mathrm{H}}\mathbf{R}_{l,(t-1)}^{-1}$ in (11) and $\lambda_{l,(t)}$ in (12), which is $O(L^2)$. Consequently, we

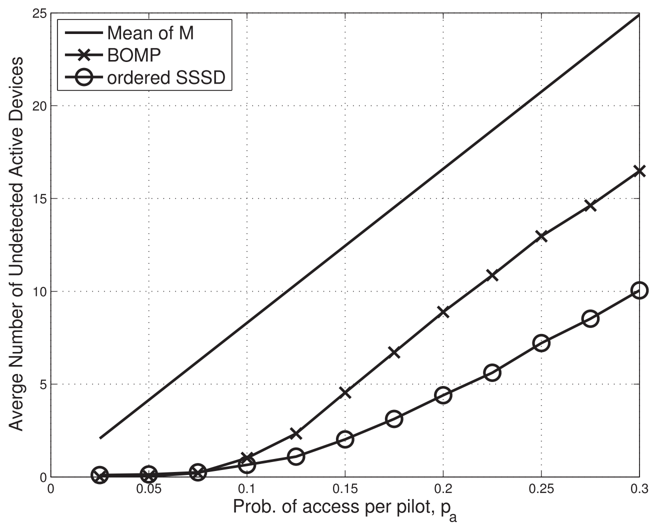
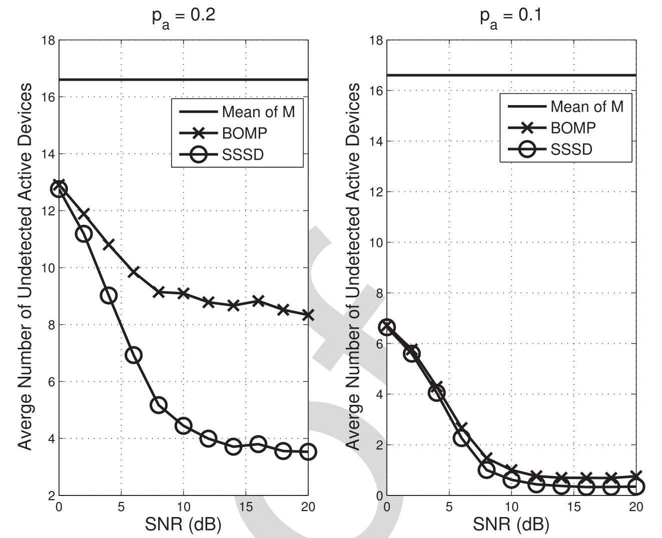
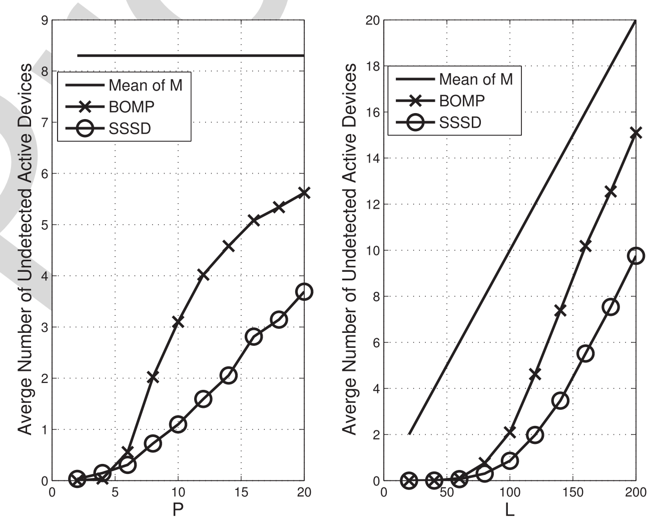


Fig. 1. Average number of undetected active devices for various values of p_a when $N = 83$, $L = N$, $P = 6$, and SNR $= 10$ dB.



(a) (b)

Fig. 2. Average number of undetected active devices for different SNRs when $N = 83$, $L = N$, and $P = 6$. (a) $p_a = 0.2$; (b) $p_a = 0.1$.



(a) (b)

Fig. 3. Performance of BOMP and ordered SSSD when $N = 83$, $p_a = 0.1$, and SNR $= 20$ dB. (a) Average number of undetected active devices versus P with $L = N$. (b) Average number of undetected active devices versus L with $P = 6$.

can see that the complexity of the ordered SSSD approach is comparable to that of the BOMP algorithm.

V. Simulation Results

In this section, we present simulation results for random access when $\mathbf{s}_l = \mathbf{a}_l u_l$. We consider A1) and A2) to generate the channel vectors with $\mathbb{E}[||\mathbf{a}_l||^2] = 1$. Thus, we consider Rayleigh multipath fading channels. For $\mathcal{C} = \{\mathbf{c}_l\}$, we use Alltop sequences in (4). In addition, we assume $\Pr(u_l = 1) = p_a$ for all l, where p_a is the probability of access per device. The signal-to-noise ratio (SNR) is defined as $\mathrm{SNR} = \frac{\mathbb{E}[||\mathbf{a}_l||^2]}{N_0} = \frac{1}{N_0}$. For performance comparisons, we consider the BOMP algorithm with known M. Note that in [7], the BOMP algorithm is employed for CS-based MUD.

Fig. 1 shows the average number of undetected active devices for various values of p_a when $N = 83$, $L = N$, $P = 6$, and SNR $= 10$ dB. Note that the mean of M grows linearly with p_a as $\mathbb{E}[M] = Lp_a$. The ordered SSSD approach outperforms BOMP for all p_a.

Fig. 2 shows the average number of undetected active devices for different SNRs when $N = 83$, $L = N$, $P = 6$, and $p_a \in \{0.1, 0, 2\}$. We can observe that when p_a is low, the BOMP algorithm and the SSSD approach perform well at high SNR. However, when p_a is not low (e.g., $p_a = 0.2$), the SSSD approach can provide a reasonably good performance (i.e., a small number of undetected active devices) at high SNR, but the BOMP algorithm cannot.

In Fig. 3(a) and (b), we show the average number of undetected active devices for various values of P (with fixed $L = N$) and L (with fixed $P = 6$), respectively, when $N = 83$, $p_a = 0.1$, and SNR $= 20$ dB. We can see that the performance is degraded as P increases in both BOMP and (ordered) SSSD. While the number of undetected active devices can increase with L for fixed p_a, in Fig. 3(b), we see that the growth rate of the average number of undetected active devices of SSSD is slower than that of BOMP. In any cases, we can see that the SSSD approach outperforms the BOMP algorithm. In particular, for a large P (e.g., $P = 10$), the SSSD approach can be about three times better than the BOMP algorithm in terms of the average number of undetected active devices.

VI. Concluding Remarks

In this letter, we proposed an approach for sparse signal recovery based on successive hypothesis testing. The proposed approach was applied to random access to detect multiple signals from active devices and to estimate their CIRs over frequency-selective fading channels. In the proposed approach, i.e., the ordered SSSD approach, since the matching pursuit has been carried out for both active and inactive signals in the ordered decision, we could achieve a better than the BOMP algorithm. We also showed that the complexity of the ordered SSSD approach is comparable to that of the BOMP algorithm.

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