Power Allocation for Max-Sum Rate and Max-Min Rate Proportional Fairness in NOMA

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Abstract—**In this letter, we consider proportional fairness scheduling (PFS) for downlink non-orthogonal multiple access (NOMA) with two users. Unlike conventional multiuser diversity systems, the power allocation plays a key role in PFS for NOMA as it can support multiple users simultaneously with positive transmission rates. With different criteria, we find optimal power allocation. Among those, it is shown that the PFS scheme that maximizes the minimum normalized rate can provide not only proportional fairness, but also small variation of transmission rates.**

Index Terms—**Nonorthogonal multiple access, proportional fairness, power allocation.**

I. INTRODUCTION

IN [1], multiuser diversity is considered to maximize the throughput of the downlink channel of a multiuser system by allocating the channel to the user of the highest channel gain. Since the users of weak channel gains may not be able to access the channel, a scheduler can be used for fairness at the cost of degraded throughput. In order to exploit the trade-off between the throughput and fairness, proportional fairness scheduling (PFS) in [2] has been studied in [3]. In [4], the performance of PFS is analyzed.

While multiuser diversity can be seen as an extreme scheduler that only maximizes the throughput, there is another extreme one that only pursue fairness. In this extreme case, the total downlink channel can be divided into multiple orthogonal channels in the time or frequency domain and allocate them to multiple users, which results in time division multiple access (TDMA) or frequency division multiple access (FDMA), respectively.

Recently, non-orthogonal multiple access (NOMA) is extensively studied in [5]–[9] as it can provide a higher spectral efficiency than orthogonal multiple access. There is a close relationship between NOMA and multiuser diversity. In NOMA, the power allocation between users who are allocated in the same radio resource block is crucial to maximize the spectral efficiency. If the power allocation maximizes the spectral efficiency without any constraints, NOMA becomes multiuser diversity where only the user of the highest channel gain is supported with a total channel. Thus, in the power allocation within NOMA, the fairness issue has to be considered to avoid the case where the users of weak channel gains cannot be supported, while providing a higher spectral efficiency than OMA.

It is noteworthy that unlike multiuser diversity, NOMA is able to support multiple users simultaneously. That is, all the users allocated in the same resource block can be simultaneously supported with certain positive rates. Without losing this salient feature, in this letter, we derive PFS for NOMA with two users under two different criteria: the maximum of the sum rate and the maximum of the minimum rate. We observe that the variation of transmission rates can be small when a PFS scheme for NOMA that maximizes the minimum rate is employed.

Notation: Matrices and vectors are denoted by upper- and lower-case boldface letters, respectively. $\mathbf{I}$ stands for the identity matrix. $\mathbb{E}[\cdot]$ and $\text{Var}(\cdot)$ denote the statistical expectation and variance, respectively. $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ represents the distribution of circularly symmetric complex Gaussian (CSCG) random vector with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{R}$.

II. SYSTEM MODELS

In this section, we consider downlink transmissions for two[1] users allocated in the same radio resource block and present the system models of NOMA and PFS for multiuser diversity.

A. NOMA

Denote by $h_k(t)$ the channel coefficient from a base station (BS) to user k during slot t. Throughout the letter, we assume block fading channels. In each slot t, we assume that the BS transmits a coded signal block to user k, denoted by $\mathbf{s}_k(t)$. In NOMA, we consider superposition coding to transmit signals to the two users. At user k, the received signal is given by

$$\mathbf{r}_k(t) = h_k(t)\mathbf{s}(t) + \mathbf{n}_k, \ \ k = 1, 2, \tag{1}$$

where $\mathbf{n}_k(t) \sim \mathcal{CN}(0, \mathbf{I})$ is the independent background noise at user k and $\mathbf{s}(t) = \mathbf{s}_1(t) + \mathbf{s}_2(t)$. Suppose that user 1 is close to the BS, while user 2 is far away from the BS. As a result, we expect that the channel gain to user 1 is higher than that to user 2. Thus, user 1 can decode the signal to user 2, $\mathbf{s}_2(t)$, first and remove it, which is called successive interference cancellation (SIC). After SIC, user 1 can decode the desired signal, $\mathbf{s}_1(t)$. At user 2, the signal to user 1 is assumed to be an interfering signal. Denote by $R_k(t)$ the achievable rate to user k at slot t. Then, under the assumption that capacity achieving Gaussian codes are employed (where $\mathbf{s}_k(t) \sim \mathcal{CN}(0, P_k\mathbf{I})$ is a

[1]While we only focus on the case of two users in this letter, it might be possible to extend to the case of more than 2 users with beamforming or multiple input multiple output (MIMO) systems.

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codeword), as in [6], [7], and [9], the achievable rates can be found as

$$R_1(t) = \log_2(1 + \alpha_1(t)P_1)$$
$$R_2(t) = \log_2\left(1 + \frac{\bar{\alpha}_2(t)P_2}{\bar{\alpha}_2(t)P_1 + 1}\right), \tag{2}$$

where $\bar{\alpha}_2(t) = \min\{\alpha_1(t), \alpha_2(t)\}$ and $\alpha_k(t) = |h_k(t)|^2$, which is the channel (power) gain to user k at slot t.

B. PFS for Multiuser Diversity

An opportunistic time-sharing orthogonal multiple access based on multiuser diversity can be employed when two (or more than two) users share a common radio resource block. To maximize the throughput, the user of the highest channel gain can be chosen as follows [1]:

$$k^*(t) = \operatorname*{argmax}_k \eta_k(t), \tag{3}$$

where $\eta_k(t) = \log_2(1 + \alpha_k(t)P_T)$ and P_T denotes the total transmission power. For convenience, this multiple access is referred to as multiuser diversity multiple access (MUD-MA).

While MUD-MA can maximize the throughput, the user of the weaker channel gain (say, user 2) cannot access channel. To avoid this, PFS can be used [3]. In this case, we have

$$k^*(t) = \operatorname*{argmax}_k \frac{\eta_k(t)}{\tau_k(t)}, \tag{4}$$

where

$$\tau_k(t+1) = \begin{cases} \nu\eta_k(t) + (1-\nu)\tau_k(t), & \text{if } k^*(t) = k \\ \tau_k(t), & \text{otherwise.} \end{cases} \tag{5}$$

Here, $\nu \in [0, 1]$ is a design parameter that is used to track the average throughput. According to [2], $\nu = \frac{1}{t_c}$, where t_c is the time constant for the moving average of $\eta_k(t)$, $\tau_k(t)$.

III. Power Allocation for Sum Rate and Max-Min Rate Proportional Fairness

In PFS for MUD-MA, in each time slot, only one user is chosen. Thus, a user of low channel gain may not be able to receive any data packets for a certain number of slots, which may result in a severe fluctuation of transmission rates and undesirable delay. On the other hand, since NOMA can support two users simultaneously, PFS can be considered with NOMA to achieve proportional fairness with possibly small variation of transmission rates. In some applications, it is desirable to have small variation of transmission rates. In particular, if the traffic generation rate is almost constant and the size of buffer is limited, it is highly desirable to have small variation of transmission rates, which also results in low latency. In this section, we consider PFS for NOMA and study the power allocation between two users under different criteria.

A. Power Allocation for Sum Rate Proportional Fairness

In NOMA, suppose that

$$P_1 + P_2 = P_T. \tag{6}$$

As in (2), since the two users can be simultaneously supported at positive rates, unlike PFS for MUD-MA, the power allocation could play a key role in PFS for NOMA. Using a similar approach to PFS for MUD-MA in (4), P_1 can be found for the power allocation as follows:

$$P_1^* = \operatorname*{argmax}_{P_1} \frac{R_k(t)}{T_k(t)} \quad \text{subject to } 0 \le P_1 \le P_T, \tag{7}$$

where

$$T_k(t+1) = \nu R_k(t) + (1-\nu)T_k(t), \ k = 1, 2. \tag{8}$$

Theorem 1: The problem in (7) *is reduced to that in* (4).

Proof: See Appendix A. ■

According to Theorem 1, the PFS scheme to maximize the normalized rates does not take advantage of NOMA where the powers to two users can be greater than zero to supports both the users simultaneously with positive rates. Thus, we need to consider a different objective function. For example, we can consider the sum of normalized rates as follows:

$$P_1^* = \operatorname*{argmax}_{P_1} \sum_k \frac{R_k(t)}{T_k(t)} \quad \text{subject to } 0 \le P_1 \le P_T. \tag{9}$$

Suppose that the past rates, $R_k(t-i)$, $i = 1, \ldots$, are low for user k. Then, $T_k(t)$ becomes small or the weight, i.e., $\frac{1}{T_k(t)}$, to $R_k(t)$ becomes large and the sum rate might be dominated by $\frac{R_k(t)}{T_k(t)}$. In this case, the power allocation is to be carried out to maximize the normalized rate for user k, $\frac{R_k(t)}{T_k(t)}$. Then, $R_k(t)$ might be high and a certain fairness can be achieved. From this, we can see that the maximization of the sum of normalized rates could provide fairness through the power allocation in NOMA.

Theorem 2: For convenience, we omit the time index and let $c_k = \frac{1}{T_k}$. *Then, the solution to* (9) *is given by*

$$P_1^* = \begin{cases} P_T, & \text{if } c_1 \ge c_2 \\ \min\{P_T, \bar{P}_1\}, & \text{if } c_1 < c_2 \text{ and } \alpha_1 c_1 > \bar{\alpha}_2 c_2 \\ 0, & \text{if } c_1 < c_2 \text{ and } \alpha_1 c_1 \le \bar{\alpha}_2 c_2, \end{cases} \tag{10}$$

where $\bar{P}_1 = \frac{c_1\alpha_1 - c_2\bar{\alpha}_2}{(c_2 - c_1)\alpha_1\bar{\alpha}_2}$.

Proof: See Appendix B. ■

The PFS scheme for NOMA to maximize the sum rate in (9) can be generalized to the case of more than two users. Unfortunately, in this case, it is not easy to find a closed-form solution. Furthermore, since the objective function[2] is not concave, the existence of a unique solution is unknown.

B. Power Allocation for Max-Min Rate Proportional Fairness

For more proactive fairness in NOMA, we can consider the maximization of the minimum normalized rate as follows:

$$P_1^* = \operatorname*{argmax}_{P_1} \min_k \frac{R_k(t)}{T_k(t)} \quad \text{subject to } 0 \le P_1 \le P_T. \tag{11}$$

The power allocation according to (11) is to achieve $\frac{R_1(t)}{T_1(t)} \approx \frac{R_2(t)}{T_2(t)}$. If $T_k(t) \to T_k > 0$, we expect to have $R_1(t) \approx \frac{T_1}{T_2} R_2(t)$ for all t. Thus, the variation of $R_k(t)$ over

[2]The objective function is concave when $T_k(t)$ is the same for all k. However, this is not the case in this letter as $T_k(t)$ is different.

the time could be small as each user has a non-zero[3] rate. It is possible to have this feature because the two users can be simultaneously supported in NOMA.

Since it is difficult to find a solution to (11), we consider an approximation. For a large $B > 0$, we can have the following approximation:

$$B^{-\min_k \frac{R_k(t)}{T_k(t)}} \approx \sum_k B^{-\frac{R_k(t)}{T_k(t)}}. \quad (12)$$

Using (12) and noting that B^{-x} is a decreasing function of $x \geq 0$, (11) can be modified as

$$P_1^* = \underset{P_1}{\operatorname{argmin}} \sum_k B^{-\frac{R_k(t)}{T_k(t)}} \\ \text{subject to } 0 \leq P_1 \leq P_{\mathrm{T}}. \quad (13)$$

Theorem 3: For convenience, we omit the time index t. The objective function in (13) *is convex and has a unique solution if $n_1 \geq 0$ and $n_2 \geq 1$, where*

$$n_k = \frac{\log_2 B}{T_k}, \ k = 1, 2. \quad (14)$$

Proof: See Appendix C. ■

Note that since the objective function in (13) is convex, it is easy to find the optimal power using a convex optimization solver.

IV. Simulation Results

In this section, we present simulation results for Rayleigh fading channels where $h_k(t) \sim \mathcal{CN}(0, \sigma_k^2)$. Furthermore, we assume that $h_k(t)$ is independent (i.e., block fading channels are assumed). In order to see the variation of the transmission rates, we consider the following normalized standard deviation (STD):

$$\text{Normalized STD} = \frac{\sqrt{\mathrm{Var}(R_k(t))}}{\mathbb{E}[R_k(t)]}.$$

If the normalized STD is small, we can see that the variation of transmission rate is small, which is desirable for low latency. In addition, the queue or buffer size can be small in this case.

For convenience, we assume that $\sigma_1^2 = 1$ (i.e., the channel power gain of user 1 is normalized). The normalized distance between the BS and user 2 is denoted by d (assuming that the distance between the BS and user 1 is set to 1). Thus, we have $\sigma_2^2 = \frac{1}{d^\kappa}$, where κ is the path loss exponent. We assume that $\kappa = 3.5$ in this section.

For performance comparisons, we consider the PFS schemes in (4) (or (7)), (9), and (13), which are referred to as MUD, NOMA/Sum, and NOMA/Max-Min, respectively, for convenience. For NOMA/Max-Min, we assume that $B = 32$.

Fig. 1 shows the performances of the PFS schemes for MUD-MA and NOMA for different values of the total transmission power, P_{T}, when $d = 2$ and $\nu = 0.9$. We can see that the transmission rates of the three PFS schemes are similar to each other, while MUD provides the best performance in terms of the sum rate. On the other hand, we can observe that NOMA/Max-Min provides the smallest normalized STD of rates.

[3]If $R_2(t)$ is zero, $R_1(t)$ has to be zero. Thus, each user should have a non-zero rate in this power allocation.

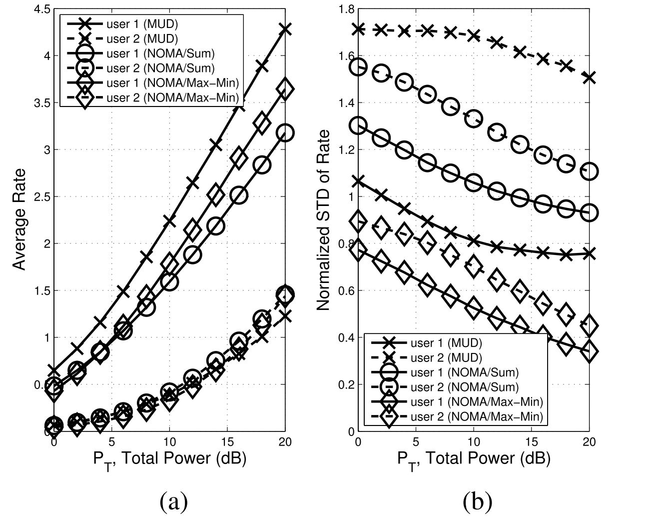

Fig. 1. Performances of the three PFS schemes for different values of the total power, P_{T} when $d = 2$ and $\nu = 0.9$: (a) transmission rates; (b) normalized STD of rates.

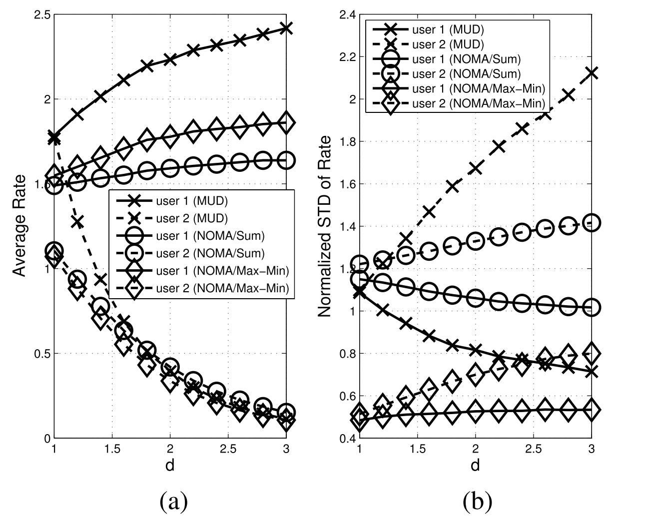

Fig. 2. Performances of the three PFS schemes for different values of d when $P_{\mathrm{T}} = 10$ dB and $\nu = 0.9$: (a) transmission rates; (b) normalized STD of rates.

Note that if TDMA is used to support two users, the average achievable rates to users 1 and 2 (with $d = 2$) are 2.942 and 1.382, respectively, when $P_{\mathrm{T}} = 20$ dB. The average transmission rates of NOMA/Max-Min to users 1 and 2 are 3.644 and 1.439, respectively, when $P_{\mathrm{T}} = 20$ dB, according to Fig. 1. From this, we can see that NOMA with PFS can still provide higher transmission rates than OMA.

Fig. 2 shows the performances of the PFS schemes for MUD-MA and NOMA for various values of d when $P_{\mathrm{T}} = 10$ dB and $\nu = 0.9$. As d increases, the channel gain of user 2 becomes lower and the transmission rate decreases (on the other hand, the transmission rate of user 1 can increase) in all the PFS schemes. We note that MUD has the highest sum rate, while it has widely varying transmission rates (to user 1). On the other hand, NOMA/Max-Min provides small normalized STDs of rates. It is noteworthy that the sum rate of NOMA/Max-Min is higher than that of NOMA/Sum. Although it is not shown in this letter, we can see the case that the sum rate of NOMA/Sum can be higher than that of

NOMA/Max-Min when ν is small (i.e., when the time constant for the moving average is large).

Overall, we can confirm that the variation of transmission rates can be small if NOMA/Max-Min is employed at the cost of degraded sum rate (compared with the PFS scheme for MUD). That is, NOMA/Max-Min can not only provide proportional fairness, but also stabilize transmission rates by supporting two users simultaneously as shown in Figs. 1 and 2.

V. Concluding Remarks

We studied NOMA with fairness and showed that the power allocation plays a crucial role in PFS for NOMA. For the power allocation, we considered different criteria. Among those, we found that the criterion to maximize the minimum normalized rate can lead to a PFS scheme of good properties. In particular, the resulting scheme can provide not only proportional fairness, but also small variation of transmission rates. For further research topics, we may consider a generalization of PFS for NOMA with more than two users and performance analysis for randomly distributed users.

Appendix A
Proof of Theorem 1

For convenience, we omit the time index t. Since $P_1 + P_2 = P_{\rm T}$, we can show that

$$\frac{\log_2(1+\alpha_1 P_1)}{T_1} \le \frac{\log_2(1+\alpha_1 P_{\rm T})}{T_1}$$
$$\frac{\log_2\left(1+\frac{\bar{\alpha}_2 P_2}{\bar{\alpha}_2 P_1+1}\right)}{T_2} \le \frac{\log_2(1+\bar{\alpha}_2 P_{\rm T})}{T_2}. \quad (15)$$

Thus, it can be further shown that

$$\max_{P_1} \frac{R_k}{T_k} = \max_{P_1} \left\{ \frac{\log_2(1+\alpha_1 P_1)}{T_1}, \frac{\log_2\left(1+\frac{\bar{\alpha}_2 P_2}{\bar{\alpha}_2 P_1+1}\right)}{T_2} \right\}$$
$$= \max \left\{ \frac{\log_2(1+\alpha_1 P_{\rm T})}{T_1}, \frac{\log_2(1+\bar{\alpha}_2 P_{\rm T})}{T_2} \right\}. \quad (16)$$

This implies that the optimal power allocation is either $P_1^* = 0$ or $P_1^* = P_{\rm T}$, which is the power allocation for MUD-MA. This completes the proof.

Appendix B
Proof of Theorem 2

For the proof, we need to see the properties of the objective function of the problem in (9), which is given by

$$\sum_k \frac{R_k}{T_k} = \frac{1}{\ln 2}\big(c_1 \ln(1+\alpha_1 P_1) - c_2 \ln(1+\bar{\alpha}_2 P_1)$$
$$+ c_2 \ln(1+\bar{\alpha}_2 P_T)\big).$$

The first derivative of the objective function becomes

$$\frac{d}{dP_1}\sum_k \frac{R_k}{T_k} = \frac{1}{\ln 2}\left(\frac{\alpha_1 c_1}{1+\alpha_1 P_1} - \frac{\bar{\alpha}_2 c_2}{1+\bar{\alpha}_2 P_1}\right)$$
$$= \frac{1}{\ln 2}\left(\frac{c_1\alpha_1 - c_2\bar{\alpha}_2 + P_1(c_1-c_2)\alpha_1\bar{\alpha}_2}{(1+\alpha_1 P_1)(1+\bar{\alpha}_2 P_1)}\right).$$

Since $\alpha_1 \ge \bar{\alpha}_2$, if $c_1 \ge c_2$, we see that $\frac{d}{dP_1}\sum_k \frac{R_k}{T_k} \ge 0$, i.e., the objective function is a nondecreasing function of P_1. Thus, $P_1^* = P_{\rm T}$.

Now suppose that $c_1 < c_2$. Then, the numerator of $\frac{d}{dP_1}\sum_k \frac{R_k}{T_k}$ decreases with P_1. If $\alpha_1 c_1 \le \bar{\alpha}_2 c_2$, $\frac{d}{dP_1}\sum_k \frac{R_k}{T_k} \le 0$, which means that the objective function is a nonincreasing function of P_1. Thus, $P_1^* = 0$. On the other hand, if $\alpha_1 c_1 > \bar{\alpha}_2 c_2$, $\frac{d}{dP_1}\sum_k \frac{R_k}{T_k}$ is greater than 0 when $P_1 = 0$, and it becomes 0 when the numerator is 0 or $P_1 = \bar{P}_1$. As P_1 is greater than $\bar{P}_1$, $\frac{d}{dP_1}\sum_k \frac{R_k}{T_k} < 0$. That is, the objective function increases with P_1 and becomes the maximum when $P_1 = \bar{P}_1$, and then decreases. Since $P_1 \le P_{\rm T}$, we have

$$P_1 = \min\{P_{\rm T}, \bar{P}_1\}.$$

This completes the proof.

Appendix C
Proof of Theorem 3

It can be shown that $B^{-\frac{R_1}{T_1}} = B^{-\frac{1}{T_1}\log_2(1+\alpha_1 P_1)} = (1+\alpha_1 P_1)^{-n_1}$. In addition, it is straightforward to show that

$$B^{-\frac{R_2}{T_2}} = B^{-\frac{1}{T_2}\left(\log_2(1+\bar{\alpha}_2 P_{\rm T}) - \log_2(1+\bar{\alpha}_2 P_1)\right)}$$
$$= W_2(1+\bar{\alpha}_2 P_1)^{n_2}, \quad (17)$$

where $W_2 = B^{-\frac{\log_2(1+\bar{\alpha}_2 P_{\rm T})}{T_2}} = (1+\bar{\alpha}_2 P_{\rm T})^{-n_2}$. Then, (13) can be rewritten as

$$P_1^* = \underset{P_1}{\operatorname{argmin}} (1+\alpha_1 P_1)^{-n_1} + W_2(1+\bar{\alpha}_2 P_1)^{n_2}$$
$$\text{subject to } 0 \le P_1 \le P_{\rm T}. \quad (18)$$

For any $n_1 > 0$, $(1+\alpha_1 P_1)^{-n_1}$ is convex in P_1 as its second derivative is non-negative. In addition, it is also easy to show that $(1+\bar{\alpha}_2 P_1)^{n_2}$ is convex in P_1 for any $n_2 \ge 1$. Since $W_2 > 0$, the cost function in (18) is a convex function of P_1. Consequently, the cost function has a unique minimum.

References

[1] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE ICC*, Jun. 1995, pp. 331–335.

[2] A. Jalali, R. Padovani, and R. Pankaj, "Data throughput of CDMA-HDR a high efficiency-high data rate personal communication wireless system," in *Proc. IEEE 51st Veh. Technol. Conf. (VTC) Spring*, vol. 3. Tokyo, Japan, May 2000, pp. 1854–1858.

[3] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, Jun. 2002.

[4] J.-G. Choi and S. Bahk, "Cell-throughput analysis of the proportional fair scheduler in the single-cell environment," *IEEE Trans. Veh. Technol.*, vol. 56, no. 2, pp. 766–778, Mar. 2007.

[5] B. Kimy *et al.*, "Non-orthogonal multiple access in a downlink multiuser beamforming system," in *Proc. IEEE Military Commun. Conf. (MILCOM)*, Nov. 2013, pp. 1278–1283.

[6] J. Choi, "Non-orthogonal multiple access in downlink coordinated two-point systems," *IEEE Commun. Lett.*, vol. 18, no. 2, pp. 313–316, Feb. 2014.

[7] Z. Ding, Z. Yang, P. Fan, and H. V. Poor, "On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users," *IEEE Signal Process. Lett.*, vol. 21, no. 12, pp. 1501–1505, Dec. 2014.

[8] K. Higuchi and A. Benjebbour, "Non-orthogonal multiple access (NOMA) with successive interference cancellation for future radio access," *IEICE Trans. Commun.*, vol. E98-B, no. 3, pp. 403–414, Mar. 2015.

[9] Q. Sun, S. Han, C.-L. I, and Z. Pan, "On the ergodic capacity of MIMO NOMA systems," *IEEE Wireless Commun. Lett.*, vol. 4, no. 4, pp. 405–408, Aug. 2015.