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Analog Beamforming for Low-Complexity Multiuser Detection in mm-Wave Systems

Jinho Choi

***Abstract*—We study multiuser detection for uplink transmissions using only analog beamformers in a millimeter-wave (mm-wave) communication system under a limited scattering environment in this paper. In particular, the successive interference cancelation (SIC)-based detection at a base station (BS) that is equipped with an antenna array is considered with the outputs of analog beamformers. We employ beam selection from a codebook of analog beams for low-complexity analog beamforming in the SIC-based detection and compressive sensing (CS)-based channel estimation that is suitable for the BS that only has analog beamformers. Through simulation results, we can see that the performance of the joint analog beam selection and SIC-based detection is comparable with the performance of the SIC-based detection (with ideal digital beamforming) at the cost of more antenna elements with lower complexity.**

***Index Terms*—Beam selection, millimeter-wave (mm-wave), multiuser detection.**

I. Introduction

For next-generation cellular systems (i.e., 5G), the millimeter-wave (mm-wave) band has been considered to meet the growing demand of high-data-rate services in the near future [1]–[4]. Although a wide bandwidth is available, it is well known that mm-wave channels suffer from a higher propagation loss and have much limited scattering [5], [6].

To overcome high path loss and exploit limited scattering environments, beamforming plays a significant role in mm-wave systems [3], [7]. Furthermore, beamforming can help reduce interference from adjacent cells. In [8], a coding approach is studied to provide a diversity gain with beamforming for mm-wave channels with line-of-sight (LoS) and relatively weak non-LoS (NLoS) paths.

For beamforming in mm-wave systems, it is expected to have a large number of antenna elements in an array. Although flexible beamforming in which any beamforming weight vector can be chosen would be ultimately possible in the future, there might be various implementation constraints with current technologies. Due to such implementation difficulties that are mainly related to radio-frequency (RF) hardware [9], [10], beamforming is not fully flexible even at a base station (BS). This has been the main motivation to study hybrid beamforming using analog and (limited) digital beamformers [7], [11]–[15]. As an alternative approach, beam selection can be also considered if beams are decided in advance (i.e., a set of fixed beams) and a beam or a group of beams is to be selected from a set of multiple beams or codebook. An advantage of beam selection over conventional beamforming[1] with fully optimized beams lies in implementations for beamforming in mm-wave systems. Beam selection from a codebook can be seen as a special case of hybrid beamforming where the digital beamformer is merely an analog beam selector. It is noteworthy that, for a large number of scatterers, the beam selection approach cannot provide a reasonable performance [13]–[15] (in this case, hybrid beamforming needs to be considered). Thus, the beam selection approach might be suitable under poor scattering environments.

In this paper, we study multiuser detection at a BS for uplink transmissions in mm-wave systems under poor scattering environments using a set of analog beamformers with a codebook of analog beams to lower the computational complexity. Since the BS only has RF analog beamformers, its implementation cost for its receiver can be low due to a small number of RF chains or downconverters. However, its performance could be limited. To overcome this difficulty, we propose an approach for successive interference cancelation (SIC)-based detection with a set of analog beamformers to detect multiple user signals, which is the main contribution of this paper. Note that SIC-based detection has been usually considered with digital beamforming [16]. In this paper, we show that it is possible to perform SIC with the outputs of analog beamformers only.

Notation: Uppercase and lowercase boldface letters are used for matrices and vectors, respectively. $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and Hermitian transpose of $\mathbf{A}$, respectively. The determinant of a square matrix $\mathbf{A}$ is denoted by $\det(\mathbf{A})$. For a vector $\mathbf{a}$, the kth element is represented by $[\mathbf{a}]_k$. For matrix $\mathbf{A}$, the (k, l)th element is denoted by $[\mathbf{A}]_{k,l}$. $\mathbb{R}^n$ and $\mathbb{C}^n$ represent the n-dimensional real and complex vector spaces, respectively. The statistical expectation is denoted by $\mathbb{E}[\cdot]$, and $\text{Var}(\cdot)$ denotes the variance. $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ represents the distribution of circularly symmetric complex Gaussian random vectors with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{R}$, where the covariance matrix of a random vector $\mathbf{x}$ is given by $\text{Cov}(\mathbf{x}) = \mathbb{E}[(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^H]$. For set $\mathcal{S}$, $|\mathcal{S}|$ stands for the cardinality of $\mathcal{S}$.

Manuscript received April 10, 2015; revised June 28, 2015 and September 7, 2015; accepted September 12, 2015. Date of publication September 17, 2015; date of current version August 11, 2016. This work was supported by the "Basic Research Projects in High-Tech Industrial Technology" Project through a grant provided by Gwangju Institute of Science and Technology in 2015. The review of this paper was coordinated by Prof. R. Dinis.

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Digital Object Identifier 10.1109/TVT.2015.2479632

[1]Clearly, conventional beamforming with fully optimized beams can offer optimal performances and outperforms beam selection at the expense of high implementation costs.

II. System Model

Suppose that a multiuser system consists of a BS and K users. The BS is equipped with an array of L antenna elements for receive beamforming, and each user has a single antenna. In this paper, we consider uplink transmissions over mm-wave channels. As the carrier frequency is high (i.e., a few tens of gigahertz), the antenna array can have a number of antenna elements of a physically limited size [9], [17]–[19]. Thus, we assume that $L \gg K$. Throughout this paper, we assume that the BS only has analog beamformers with a limited number of RF chains.

A. Array Channels Under Limited Scattering

Throughout this paper, we consider a 1-D array at the BS for the sake of simplicity. Let $\mathbf{a}(\theta)$ be the array response vector (ARV) of the BS array with the angle of arrival (AoA), $\theta \in \Theta$, where Θ is the domain of the AoA. Denote by $\mathbf{h}_k$ the channel vector from user k to the BS. Then, the (narrow-band) channel vector is given by [7]

$$\mathbf{h}_k = \sum_{p=1}^{P} \alpha_{k,p} \mathbf{a}(\theta_{k,p}) \tag{1}$$

where P is the number of paths, and $\alpha_{k,p}$ and $\theta_{k,p}$ are the channel coefficient and the AoA of the signal through the pth path, respectively. Under limited scattering environments of mm-wave channels, P is usually small (2 or 3) [6]. If we assume a uniform linear array with half-wavelength spacing, the ARV is $\mathbf{a}(\theta) = [1 \; e^{-j\pi \sin\theta}, \ldots, \; e^{-j(L-1)\pi \sin\theta}]^T$, where $j = \sqrt{-1}$. Then, by letting $z_{k,p} = \pi \sin\theta_{k,p}$, it can be shown that $[\mathbf{h}_k]_l = h_{k,l} = \sum_{p=1}^{P} \alpha_{k,p} e^{-j(l-1)z_{k,p}}$.

It is noteworthy that the channel model in (1) can be also used when users are equipped with antenna arrays and employ transmit beamforming. To see this, suppose that $\mathbf{b}(\psi_{k,p})$ denotes the ARV at user k with angle of departure $\psi_{k,p}$ to the pth path. In addition, let $\mathbf{v}_k$ denote the transmit beamforming vector at user k. Then, the channel vector $\mathbf{h}_k$ in (1) is modified as

$$\mathbf{h}_k = \sum_{p=1}^{P} \alpha_{k,p} \mathbf{a}(\theta_{k,p}) \mathbf{b}^T(\theta_{k,p}) \mathbf{v}_k = \sum_{p=1}^{P} \tilde{\alpha}_{k,p} \mathbf{a}(\theta_{k,p})$$

where $\tilde{\alpha}_{k,p} = \alpha_{k,p} \mathbf{b}^T(\theta_{k,p}) \mathbf{v}_k$ becomes a new channel coefficient for the pth path. Thus, we can see that, for a given user's beamforming vector $\mathbf{v}_k$, the channel vector in (1) is valid.

B. Uplink Training

For coherent detection, the BS needs to estimate the channel vectors $\{\mathbf{h}_k\}$ using uplink training. Throughout this paper, we consider a two-slot structure for uplink transmission consisting of the uplink training and the data transmission blocks, as shown in Fig. 1. The length of the two slots should be less than a coherence time so that the estimated channels from the uplink training can be used for coherent detection of the data symbols within the following data transmission block.

Suppose that pilot signals are to be transmitted prior to transmitting data symbols, i.e., during the uplink training block. Let $\vec{\tau}_k = \{\tau_{k,t}, \; t = 0, \ldots, T-1\}$ be the uplink training sequence from user k to the BS. Here, T is the length of the uplink training sequence, which would be a fraction of the coherence time. The determination of T to increase the throughput can be found in [20]. It is assumed that the K uplink training sequences are orthogonal to each other. During the uplink training session, the users transmit the training signals to the BS. The BS receives the following signals:

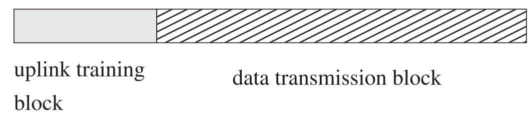

Fig. 1. Uplink transmissions consisting of uplink training and data transmission blocks.

$$\mathbf{x}_t = \sum_{k=1}^{K} \mathbf{h}_k \tau_{k,t} + \mathbf{n}_t, \quad t = 0, \ldots, T-1. \tag{2}$$

Let $P_\tau = \sum_{t=0}^{T-1} |\tau_{k,t}|^2 = \|\vec{\tau}_k\|^2$. To estimate $\mathbf{h}_k$, we can consider the output of the correlator as follows:

$$\begin{aligned}\mathbf{z}_k &= \sum_{t=0}^{T-1} \tau_{k,t}^* \mathbf{x}_t = P_\tau \mathbf{h}_k + \mathbf{u}_k \\ &= P_\tau \sum_{p=1}^{P} \alpha_{k,p} \mathbf{a}(\theta_{k,p}) + \mathbf{u}_k \end{aligned} \tag{3}$$

where $\mathbf{u}_k = \sum_{t=0}^{T-1} \tau_{k,t}^* \mathbf{n}_t$. Then, it is easy to show that $\mathbf{u}_k \sim \mathcal{CN}(0, N_0 P_\tau)$.

III. Channel Estimation Using Compressive Sensing

Since we assume that the BS only has (a small number of) analog beamformers, the conventional channel estimation methods (e.g., the linear minimum mean squared error approach [21]) may not be applicable due to limited flexibility of analog beamformers. Fortunately, however, as shown in [22], it is possible to estimate the channel vector $\mathbf{h}_k$ under a limited scattering environment using analog beamformers by exploiting the notion of compressive sensing (CS) [23]–[26]. Here, based on the CS-based approach in [22], which has been proposed for the downlink channel estimation, we consider the channel estimation for uplink transmissions.

Let $\mathcal{W}$ denote the codebook of analog beams that can be formed by an analog beamformer. We assume that $|\mathcal{W}| = N$, i.e., $\mathcal{W} = \{\mathbf{w}_{(1)}, \ldots, \mathbf{w}_{(N)}\}$, where $\mathbf{w}_{(n)}$ denotes the nth analog beamforming vector (or analog beam). It is assumed that $\mathbf{w}_{(n)} = \mathbf{a}(\theta_{(n)})$, where $\theta_{(n)}$ is the nth quantized AoA in Θ. Define $\mathbf{W} = [\mathbf{a}(\theta_{(1)}), \ldots, \mathbf{a}(\theta_{(N)})]$, which is an $L \times N$ matrix. If N is sufficiently large, there might be a quantized AoA that can approximate $\theta_{k,p}$ closely. Thus, $\mathbf{h}_k$ can be approximated by a linear combination of P vectors out of $\{\mathbf{a}(\theta_{(1)}), \ldots, \mathbf{a}(\theta_{(N)})\}$. That is, $\mathbf{W}\mathbf{c}_k \approx \mathbf{h}_k$, where $\mathbf{c}_k$ is a P-sparse signal. Then, $\mathbf{z}_k$ in (3) becomes

$$\mathbf{z}_k \approx \mathbf{W}\mathbf{c}_k + \mathbf{u}_k.$$

This approximation might be good for a large number of the quantized AoAs or N. As shown in Section IV-A, it is important to approximate $\mathbf{h}_k$ as a linear combination of vectors in $\mathbf{W}$ for low-complexity beam selection.

From a subset or subvector of $\mathbf{z}_k$, it is possible to estimate P-sparse signals exploiting the notion of CS. Denote by $\tilde{\mathbf{z}}_k$ a subvector of $\mathbf{z}_k$, and the number of elements of $\tilde{\mathbf{z}}_k$ is $J \le L$. For convenience, $\mathcal{J}$ denotes the index set of the selected J elements in $\mathbf{z}_k$ for $\tilde{\mathbf{z}}_k$ of size $J \times 1$. Denote by $\mathbf{\Psi}$ a submatrix of $\mathbf{W}$ obtained by taking the row vectors of the indices in $\mathcal{J}$. The size of $\mathbf{\Psi}$ is $J \times N$, and we have

$$\tilde{\mathbf{z}}_k \approx \mathbf{\Psi}\mathbf{c}_k + \tilde{\mathbf{u}}_k$$

where $\tilde{\mathbf{u}}_k$ of size $J \times 1$ is the a subvector of $\mathbf{u}_k$ obtained by taking the elements of the indices in $\mathcal{J}$. Under the restricted isometry property (RIP) of order $2P$, it is known that, if $J \geq CP\log(N/P)$, where C is a constant and the background noise is negligible, the P-sparse signal $\mathbf{c}_k$ can be estimated by solving

$$\min_{\mathbf{c}} \|\mathbf{c}\|_1 \text{ subject to } \tilde{\mathbf{z}}_k = \mathbf{\Psi}\mathbf{c}$$

where $\|\mathbf{x}\|_1$ is the ℓ_1-norm [25], [26]. There are various approaches to solve this problem [26]. Among those, we consider the orthogonal matching pursuit (OMP) [27] in Section V for simulations (note that other approaches can be used).

In general, if some AoAs are close to each other, we can only see a single AoA and the CS-based approach may fail to estimate $\mathbf{h}_k$ as the RIP condition does not hold. For example, suppose that $P = 2$ and a channel vector is given by

$$\mathbf{h} = \alpha_1 \mathbf{a}(\theta_1) + \alpha_2 \mathbf{a}(\theta_2)$$

where α_p and θ_p are the channel coefficient and AoA of the pth path, respectively. If we assume that $\theta_1 \approx \theta_2$, we may have

$$\mathbf{h} \approx (\alpha_1 + \alpha_2)\mathbf{a}(\theta_1) \approx (\alpha_1 + \alpha_2)\mathbf{a}(\theta_2)$$

and it is shown that both are good estimates of $\mathbf{h}$. Therefore, for the channel estimation using a CS-based approach, the separation of AoAs for the RIP condition may not be a critical issue.

IV. Signal Detection Using Analog Beamformers

Here, we focus on a receiver using only analog beamforming, which might be well suited to a BS for a small cell. We first present a SIC-based detector that has two steps. In the first step, K analog beams are chosen from $\mathcal{W}$ for beamforming. In the second step, with the selected analog beams, we perform SIC-based detection. Then, we discuss the structure for the receiver that uses analog beamformers to demonstrate that its hardware cost can be low.

A. Signal Detection Using Analog Beamformers

Throughout this paper, we assume that the number of data symbols within a coherence time is M, excluding the time for uplink training. Let $s_{k,m} \in \mathcal{S}$ denote the mth data symbol transmitted from user k, where $\mathcal{S}$ denotes the signal constellation. Then, after the uplink training period, K users can transmit data symbols simultaneously. The received signal at the BS becomes

$$\mathbf{r}_m = \sum_{k=1}^{K} \mathbf{h}_k s_{k,m} + \mathbf{n}_m, \quad m = 0, \ldots, M-1 \tag{4}$$

where $\mathbf{n}_m \sim \mathcal{CN}(0, N_0\mathbf{I})$ is the background noise vector. For convenience, we will omit the time index m if there is no risk of confusion.

With analog beamformers, we can derive various approaches for signal detection. Suppose that the BS can choose the best analog beamformer $\mathcal{W}$ for each user to maximize the following signal-to-interference-plus-noise ratio (SINR):

$$\gamma_k(\mathbf{w}) = \frac{\left|\mathbf{w}^H \hat{\mathbf{h}}_k\right|^2}{\sum\limits_{q \neq k} \left|\mathbf{w}^H \hat{\mathbf{h}}_q\right|^2 + LN_0}. \tag{5}$$

With the estimated channel vector $\hat{\mathbf{h}}_k$, for each k, the BS can decide the analog beamforming vector maximizing the SINR from $\mathcal{W}$ as $\hat{\mathbf{w}}_k = \arg\max_{\mathbf{w}\in\mathcal{W}} \gamma_k(\mathbf{w})$. Then, during data transmissions, from the outputs of the analog beamformers, $v_k = \hat{\mathbf{w}}_k^H \mathbf{r}$, $k = 1, \ldots, K$, the BS can detect signals as

$$\hat{s}_k = \mathrm{Dec}\left(\frac{\hat{\mathbf{w}}_k^H \mathbf{r}}{\hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_k}\right) = \mathrm{Dec}\left(\frac{v_k}{\beta_k}\right) \tag{6}$$

where $\beta_k = \hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_k$, and $\mathrm{Dec}(x)$ is the decision function of x to the nearest one out of $\mathcal{S}$, i.e., $\mathrm{Dec}(x) = \arg\min_{s\in\mathcal{S}} |x - s|^2$. This detector is referred to as the conventional detector in this paper.

The notion of SIC can be used for a better detector. Suppose that signal detection is carried out successively from user 1's signal to user K's signal. For s_1, we can find the best beam from $\mathcal{W}$ to maximize the SINR and obtain an estimate of s_1, i.e., $\hat{s}_1$. Since $\hat{s}_1$ is available, we can remove the signal from s_1 as $\mathbf{r}_{(1)} = \mathbf{r} - \hat{\mathbf{h}}_1 \hat{s}_1$. Then, we can choose the beam for s_2 that maximizes the SINR from $\mathbf{r}_{(1)}$ rather than $\mathbf{r}$. Consequently, for SIC, the BS needs to find the analog beams from $\mathcal{W}$ that maximize the SINRs after removing known interference as follows:

$$\gamma_{\mathrm{sic},k}(\mathbf{w}) = \frac{\left|\mathbf{w}^H \hat{\mathbf{h}}_k\right|^2}{\sum\limits_{q > k} \left|\mathbf{w}^H \hat{\mathbf{h}}_q\right|^2 + LN_0}. \tag{7}$$

That is, as in (7), we have $\hat{\mathbf{w}}_k = \arg\max_{\mathbf{w}\in\mathcal{W}} \gamma_{\mathrm{sic},k}(\mathbf{w})$. Clearly, $\{\hat{\mathbf{w}}_k\}$ can be found once the channel estimates $\{\hat{\mathbf{h}}_k\}$ are available (i.e., during the uplink training session). Once $\{\hat{\mathbf{w}}_k\}$ is found, signal detection during data transmissions can be carried out as

$$\hat{s}_k = \mathrm{Dec}\left(\frac{\hat{\mathbf{w}}_k^H \mathbf{r}_{(k)}}{\hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_k}\right), \quad k = 1, \ldots, K \tag{8}$$

where $\mathbf{r}_{(k)} = \mathbf{r} - \sum_{q=1}^{k-1} \hat{\mathbf{h}}_q \hat{s}_q$. It is noteworthy that, in (8), since

$$\hat{\mathbf{w}}_k^H \mathbf{r}_{(k)} = \hat{\mathbf{w}}_k^H \mathbf{r} - \sum_{q=1}^{k-1} \hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_q \hat{s}_q = v_k - \sum_{q=1}^{k-1} \beta_{k,q} \hat{s}_q \tag{9}$$

where $\beta_{k,q} = \hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_q$, we only need the outputs of analog beamformers $\{v_k\}$ to perform SIC.

To improve the performance further by mitigating error propagation, we can consider an ordered detection based on the SINR using a greedy approach. Let $\mathcal{K}_0 = \{1, \ldots, K\}$ and $n = 1$. The nth user in detection can be decided as

$$k_n = \arg\max_{k\in\mathcal{K}_n} \max_{\mathbf{w}\in\mathcal{W}} \hat{\gamma}_{\mathrm{sic},k}(\mathbf{w}) \tag{10}$$

where

$$\hat{\gamma}_{\mathrm{sic},k}(\mathbf{w}) = \frac{\left|\mathbf{w}^H \hat{\mathbf{h}}_k\right|^2}{\sum\limits_{q\in\mathcal{K}_n\setminus k} \left|\mathbf{w}^H \hat{\mathbf{h}}_q\right|^2 + LN_0}, \quad k \in \mathcal{K}_n. \tag{11}$$

Once k_n is found, the user set can be updated as $\mathcal{K}_{n+1} = \mathcal{K}_n \setminus k_n$. With $n \leftarrow n + 1$, we can find the next user as in (10). The resulting ordered SIC-based detector can provide a better performance as both the beam and user selection diversity gains are exploited.

For the signal detection, we need to obtain the SINRs in (5) or (7). To find the SINRs, we need a number of inner products between $\hat{\mathbf{h}}_k$ and $\mathbf{w} \in \mathcal{W}$. If analog beams are used, the complexity to find the inner product can be low as follows.

Property 1: For each inner product with an analog beam, the complexity (in terms of the number of complex multiplications) is $O(P)$.

Proof: The lth element of a beam in $\mathcal{W}$, denoted by $\mathbf{w}$, can be expressed as $[\mathbf{w}]_l = w_l = e^{j\psi_l}$, where $\psi_l = \angle w_l$. Then, the inner product between $\mathbf{w} \in \mathcal{W}$ and $\mathbf{h}_k$ becomes

$$\mathbf{w}^H \mathbf{h}_k = \sum_{p=1}^{P} \alpha_{k,p} \left(\sum_{l=1}^{L} e^{-j\left(\psi_l + (l-1) z_{k,p}\right)} \right). \quad (12)$$

Clearly, the number of (complex) multiplications for the inner product is P not L, whereas the number of complex additions is PL. ■

Since $P \ll L$, making use of the codebook of analog beams can significantly reduce the computational complexity.

Furthermore, in the CS-based channel estimation in Section III, we use the measurement matrix $\mathbf{W}$. Thus, the estimated channel vector $\hat{\mathbf{h}}_k$, is a linear combination of analog beams $\mathbf{a}(\theta_{(n)})$. This implies that $\mathbf{w}^H \hat{\mathbf{h}}_k$ is a weighted sum of a few inner products between the analog beams. Since the inner products between the analog beams can be computed in advance and stored, the computation for $\mathbf{w}^H \hat{\mathbf{h}}_k$ only requires P complex multiplications.

Consequently, the complexity order to obtain the SINRs to choose analog beams from $\mathcal{W}$ becomes

$$O(PN) + O(P(N-1)) + \cdots + O\left(P(N-K+1)\right) \approx O(KNP)$$

where the approximation is valid if $K \ll N$. Once the SINRs are obtained, the K analog beams can be chosen by comparing or sorting the SINRs. The computational complexity for sorting is negligible as they are based on comparisons, not complex multiplications. Thus, we can see that the complexity for K analog beam selection is $O(KNP)$.

The main advantage of analog beams is that analog beamformers, which can be implemented in the RF domain, can be used for spatial filtering [10], [18], [19]. Suppose that there are K analog beamformers. The beamforming weight vector for the kth beamformer is $\hat{\mathbf{w}}_k$, which is an analog beam. To perform SIC-based detection, we can use the K analog beamformers' outputs, which are $v_k = \hat{\mathbf{w}}_k^H \mathbf{r}$. Since these outputs can be obtained by using K RF analog beamformers in the analog domain, there is no computational complexity to find them in the digital domain.

According to (9), we need to compute $\hat{\mathbf{w}}_k^H \mathbf{r}_{(k)}$ for the signal detection. Since $\beta_{k,q} = \hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_q$ is found in computing the SINR, there is no additional computation except for the products of $v_k = \hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_q$ and $\hat{s}_q$. Thus, the computational complexity for the signal detection is $O(1 + 2 + \cdots + K - 1) = O(K^2)$. Thus, the total computational complexity becomes $O_{\text{analog}} = O(MK^2) + O(KNP) = O(K(NP + MK))$. This computational complexity can be lower for certain signal constellations (e.g., constant modulus signal constellation). For example, suppose that quadrature phase-shift keying (QPSK) is used, i.e., $\mathcal{S} = \{\pm 1 \pm j\}$. If $s_q = 1 - j$, we have $\hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_q s_q = \hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_q - j\hat{\mathbf{w}}_k^H \hat{\mathbf{h}}_q$, which does not require any multiplication. In this case, we have

$$O_{\text{analog}} = O(KNP). \quad (13)$$

For comparison purposes, we can consider the complexity of SIC with digital beamforming. Since the complexity of the inner product $\mathbf{w}^H \mathbf{h}_k$ is $O(L)$ [not $O(P)$], the complexity to obtain the SINRs becomes $O(KL)$. Furthermore, the computational complexity for the signal detection is $O(KL) + O(K^2)$. Thus, for M symbols, the total computational complexity becomes $O_{\text{digital}} = O(K((M+1)L + MK))$. From this, we can see that, for SIC, analog beamforming can have lower computational complexity than digital beamforming by a factor of

$$\frac{O_{\text{digital}}}{O_{\text{analog}}} = \frac{K\left((M+1)L + MK\right)}{K(NP + MK)}.$$

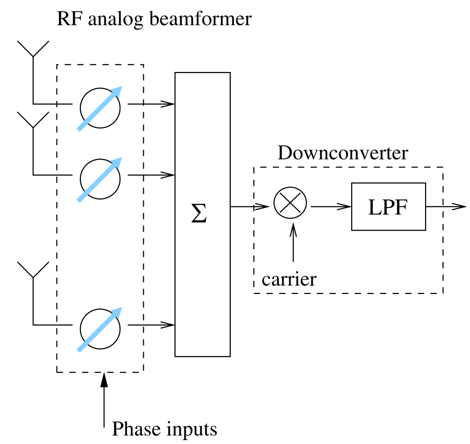

Fig. 2. Illustration of an RF analog beamformer with a downconverter.

If $M \gg L, N, K$, we have $O_{\text{digital}}/O_{\text{analog}} \approx (L + K)/K$. In addition, if a constant modulus signal constellation is used

$$\frac{O_{\text{digital}}}{O_{\text{analog}}} = \frac{K(M+1)L}{KNP} = \frac{L(M+1)}{NP}.$$

Thus, as long as $M \geq NP/L$, we can see that the SIC with analog beamforming has lower computational complexity than the SIC with digital beamforming. For example, suppose that $L = 60$, $P = 3$, and $N = 300$, the computational complexity of the SIC with analog beamforming is lower than the computational complexity of the SIC with digital beamforming if $M \geq 15$. From this, for a reasonably long coherence time, we can see that analog beamforming has not only a lower hardware cost but also much lower computational complexity than digital beamforming.

B. Analog Hardware Complexity

Here, we discuss the complexity of analog hardware for the proposed approach in terms of the required number of downconverters. For each downconverter, we need an RF chain that includes a low-noise amplifier. Since the analog beamformer can be built by passive elements [11], [28], the cost of RF chains dominates the total cost to build a system based on the proposed approach. This implies that the total cost may not be significantly high although the number of antenna elements L is large when analog beamformers are used. However, the cost becomes proportional to the number of RF chains or downconverters.

As aforementioned, for the signal detection, we need K RF analog beamformers. Each RF analog beamformer consists of L analog phase shifters and an analog mixer (summing device), as shown in Fig. 2. Thus, there should be a total of K downconverters for the signal detection.

V. Simulation Results

Here, we present simulation results with the channel vector in (1), where one path ($p = 1$) is the LoS path and the others are NLoS paths. For the channel coefficient of the LoS path, we assume that $\alpha_{k,1} = 1$. For NLoS paths, $\alpha_{k,p} \sim \mathcal{CN}(0, 1)$, $p = 2, \ldots, P$. Here, we assume that $P = 3$ (i.e., two NLoS paths). For the AoAs, $\{\theta_{k,p}\}$, we assume that they are independent uniform random variables over $[-\pi/3, \pi/3]$ (with 3-sectorization). Thus, the resulting spatial correlation of channels is high. For a codebook, the ARVs with $\{\theta_{(n)}\}$ that are uniformly quantized over $[-\pi/3, \pi/3]$ are considered.

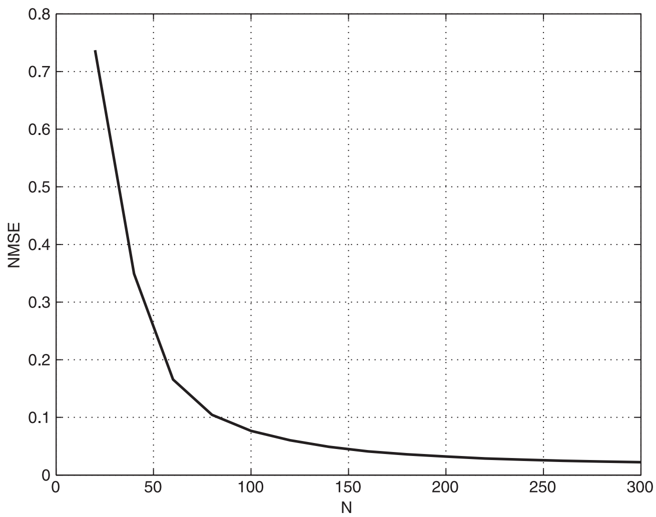

Fig. 3. NMSE versus M with $L = 60$, $J = 30$, and SNR = 10 dB.

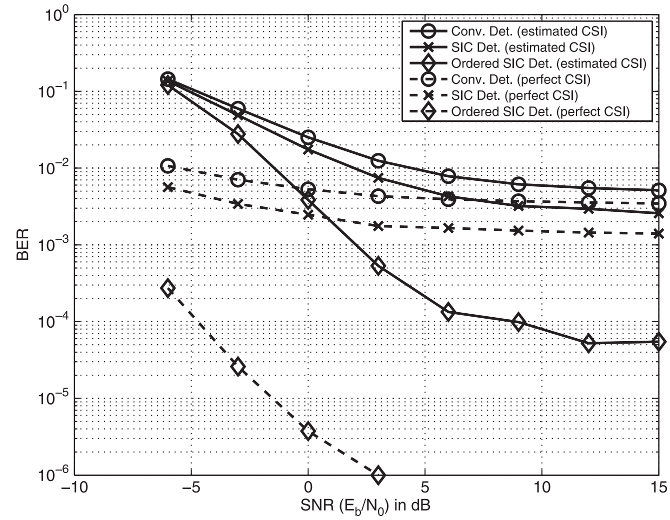

Fig. 5. BER versus SNR with $L = 60$, $N = 300$, $K = 10$, and $J = 30$.

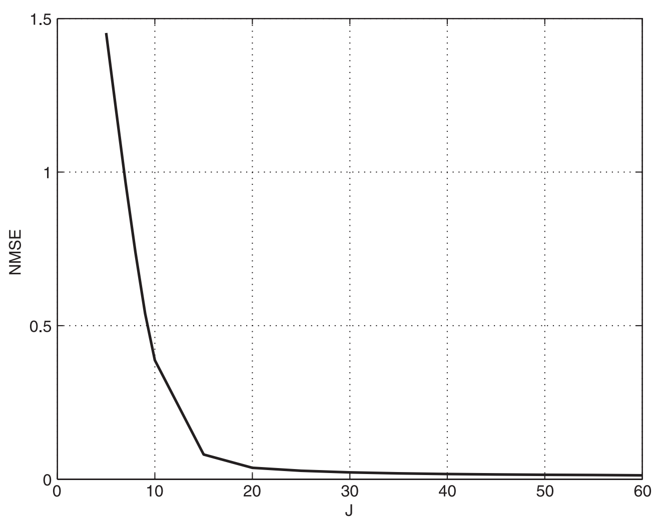

Fig. 4. NMSE versus J with $L = 60$, $M = 300$, and SNR = 10 dB.

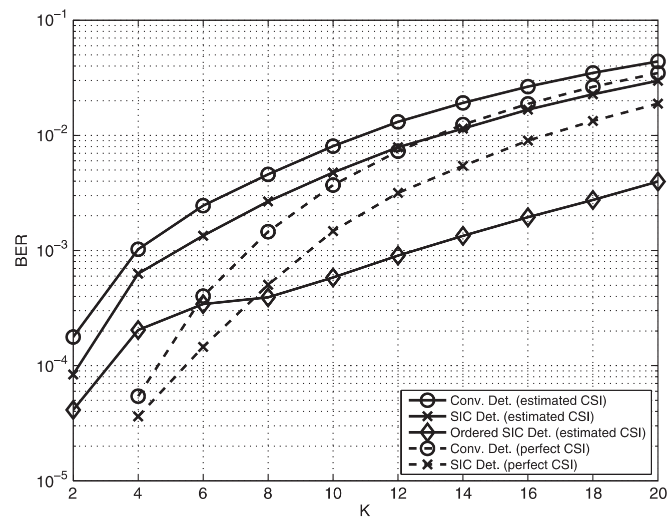

Fig. 6. BER versus K with $L = 60$, $N = 300$, SNR = 10 dB, and $J = 20$.

A. Simulation Results of Channel Estimation

To see the performance of the channel estimation, we consider the normalized mean squared error (NMSE) as follows:

$$\text{NMSE} = \frac{\mathbb{E}\left[\|\hat{\mathbf{h}} - \mathbf{h}\|^2\right]}{\mathbb{E}\left[\|\mathbf{h}\|^2\right]}.$$

For the CS-based channel estimation, we assume that J antennas are randomly chosen for each run and the OMP algorithm.

Fig. 3 shows the NMSE for different values of N when $L = 60$, $J = 30$, and SNR = 10 dB. As N increases, the CS-based channel estimation can provide a better performance as the approximation to represent $\mathbf{h}_k$ as a linear combination, i.e., $\mathbf{h}_k \approx \mathbf{W}\mathbf{c}_k$, can be improved.

Fig. 4 shows the NMSE for different values of J when $L = 60$, $N = 300$, and SNR = 10 dB. We can see that $J = 30$ seems sufficient to have reasonably small NMSE or good channel estimates.

B. Simulation Results of Signal Detection

Here, we present bit-error-rate (BER) results with estimated and perfect channel vectors or channel state information (CSI). We consider QPSK for signaling.

Fig. 5 shows the BER performances for different values of SNR, which is given by E_b/N_0, where E_b denotes the bit energy. It is assumed that $L = 60$, $N = 300$, $K = 10$, and $J = 30$. It is shown that there are error floors. In analog beam selection, since the multiuser interference cannot be perfectly suppressed, it results in an error floor. It is shown that the ordered SIC-based detector can significantly improve the error floor due to the beam and user selection diversity gains.

The impact of the number of users on the BER performance is shown in Fig. 6 when $L = 200$ and SNR = 30 dB. As expected, the BER increases with K. Note that the BER of the ordered SIC-based detector is not shown as the BER is too low (less than 10^{-5}). We can see that the performance (with estimated CSI) gap between the ordered SIC-based detector and the other detectors increases with K.

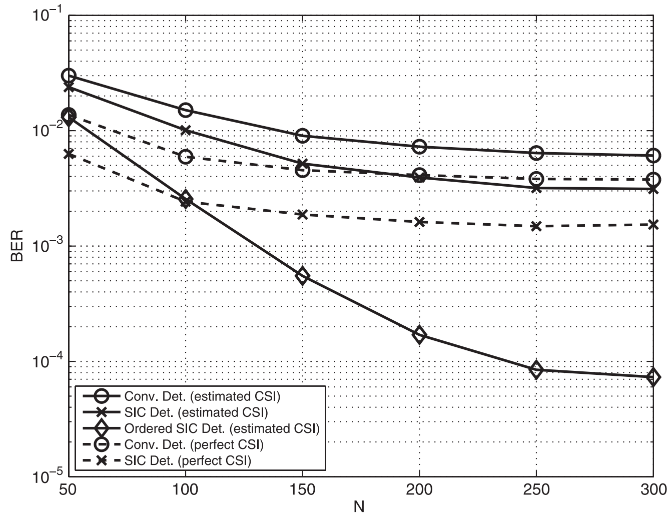

Fig. 7. BER versus N with $L = 60$, $K = 10$, SNR $= 10$ dB, and $J = 30$.

Since the ordered SIC-based detector can exploit the multiuser diversity gain, it may have less performance degradation than the other detectors.

Fig. 7 shows the BER performance for different values of N. As N increases, a better channel estimate is available, as shown in Fig. 4. Furthermore, there is a better analog beam available from selection as N increases. Thus, the BER decreases with N. It is interesting to note that the performance (with estimated CSI) gap between the ordered SIC-based detector and the other detectors increases with N.

VI. Conclusion

In this paper, we have studied analog beam selection for multiple signal detection in uplink transmissions. It was shown that analog beam selection can be performed at low computational complexity, which is independent of the number of antennas L in terms of the number of complex multiplications. Thus, in mm-wave communications where the number of antennas in an array can be large, analog beam selection can be used for multiuser signal detection to achieve reasonable performances with low computational complexity. As a further work, we will consider more details of analog implementation for the beam selection together with the channel estimation.

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