Sparse Index Multiple Access for Multi-Carrier Systems with Precoding

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Abstract: **In this paper, we consider subcarrier-index modulation (SIM) for precoded orthogonal frequency division multiplexing (OFDM) with a few activated subcarriers per user and its generalization to multi-carrier multiple access systems. The resulting multiple access is called sparse index multiple access (SIMA). SIMA can be considered as a combination of multi-carrier code division multiple access (MC-CDMA) and SIM. Thus, SIMA is able to exploit a path diversity gain by (random) spreading over multiple carriers as MC-CDMA. To detect multiple users' signals, a low-complexity detection method is proposed by exploiting the notion of compressive sensing (CS). The derived low-complexity detection method is based on the orthogonal matching pursuit (OMP) algorithm, which is one of greedy algorithms used to estimate sparse signals in CS. From simulation results, we can observe that SIMA can perform better than MC-CDMA when the ratio of the number of users to the number of multi-carrier is low.**

Index Terms: **compressive sensing, sparse index modulation, sparse index multiple access**

I. INTRODUCTION

IN [1], spatial modulation (SM) is proposed to transmit signals over multiple input multiple output (MIMO) channels with a single transmit antenna at a time. In SM, the index of the active transmit antenna is to bear information bits. Thus, if there are 4 transmit antennas, SM can transmit 2 bits per channel use. Furthermore, a modulated symbol can be transmitted by the active transmit antenna. SM can be generalized by activating more than one transmit antennas. Thus, the number of bits that can be transmitted by the indices of the active transmit antennas becomes $\lfloor \log_2 \binom{N_{\rm TX}}{K} \rfloor$, where $N_{\rm TX}$ is the number of transmit antennas and K is the number of active antennas at a time. Since a fraction of $N_{\rm TX}$ antennas are activated, SM can be energy efficient, and cost-effective as only K radio frequency (RF) chains are required. Furthermore, if $K = 1$, there is no interference from the other transmit antennas, which allows to use low-complexity detectors [1], [2]. In order to improve the performance of SM, the notion of channel coding can also be employed [3], [4].

Orthogonal frequency division multiplexing (OFDM) has been extensively studied and employed for wireless standards [5], [6] due to various advantages over other schemes. In particular, OFDM effectively mitigates inter-symbol interference (ISI) and allows to use low-complexity a one-tap equalizer for signal detection. The notion of SM can be applied to OFDM, which results in subcarrier-index modulation (SIM) OFDM [7], [8]. In SIM OFDM, a subset of subcarriers are activated and their indices are used to transmit information bits. Thus, SIM OFDM can mitigate ISI as OFDM. While SIM OFDM is energy efficient as SM, it cannot exploit path diversity, which is the same as OFDM [9].

To exploit path diversity, precoding can be used in OFDM [10]. Unfortunately, precoding can offset one of the advantages of OFDM systems, which is the orthogonality. Since the orthogonality cannot be retained due to precoding, one-tap equalizers cannot be used and more complicated equalizers or detectors are to be used to mitigate ISI. There can be certain precoding schemes for the trade-off between performance and complexity of detectors. To this end, low density spreading is devised in [11] and extended for multiple access in [12] to exploit path diversity by receivers of reasonably low complexity. In [13], SIM is considered for precoded OFDM with a small number of activated subcarriers. Due to the sparsity, the notion of compressive sensing (CS) [14], [15] can be employed to derive a low-complexity detector. CS is to recover sparse signals with a considerably low sampling rate compared to the bandwidth of observed signals. There have been various CS algorithms for the estimation of sparse signals [16]–[18]. The main advantage of a CS detector is that it can estimate SIM signals by using only a small fraction of the received signals over subcarriers. Since a small number of demodulators would be required, the cost to build a CS detector can be low.

In this paper, we consider SIM for precoded OFDM with a few activated subcarriers per user and generalize it to multi-carrier multiple access systems. The resulting system is referred to as sparse index multiple access (SIMA) in this paper. SIMA can be seen as a modification of multi-carrier code division multiple access (MC-CDMA). In SIMA, each user transmit signals using SIM with precoding over multiple carriers. At a receiver, multiple users' signals can be detected using CS algorithms as signals are sparse. In order to see that CS algorithms are applicable to signal detection, we study the restricted isometry property (RIP) [19], [20] of a composite channel matrix using Gaussian approximation. A low-complexity detector based on the orthogonal matching pursuit (OMP) algorithm [16], [21] is derived. In order to improve the detection performance, we exploit a property of sparse signals in SIMA, which results in a modified OMP algorithm for signal detection.

The main contribution of the paper is two-fold: (i) SIMA is proposed for multi-carrier systems; (ii) a CS-based low-complexity detector is proposed to detect multiple users' signals once we show that the RIP property is verified.

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Notation: Matrices and vectors are denoted by upper- and lower-case boldface letters, respectively. The superscripts T and H denote the transpose and complex conjugate, respectively. The p-norm of a vector $\mathbf{a}$ is denoted by $||\mathbf{a}||_p$ (If $p = 2$, the norm is denoted by $||\mathbf{a}||$ without the subscript). The superscript † denotes the pseudo-inverse. For a vector $\mathbf{a}$, $\text{diag}(\mathbf{a})$ is the diagonal matrix with the diagonal elements from $\mathbf{a}$. For a matrix $\mathbf{X}$ (a vector $\mathbf{a}$), $[\mathbf{X}]_n$ ($[\mathbf{a}]_n$) represents the nth column (element, resp.). If n is a set of indices, $[\mathbf{X}]_n$ is a submatrix of $\mathbf{X}$ obtained by taking the corresponding columns. $\mathbb{R}^n$ and $\mathbb{C}^n$ represent the n-dimensional real and complex vector spaces, respectively. $\mathbb{E}[\cdot]$ and $\text{Var}(\cdot)$ denote the statistical expectation and variance, respectively. $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ ($\mathcal{N}(\mathbf{a}, \mathbf{R})$) represents the distribution of circularly symmetric complex Gaussian (CSCG) (resp., real-valued Gaussian) random vectors with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{R}$.

II. SYSTEM MODELS

Throughout the paper, we consider a multi-carrier multiuser system [22], [23], which consists of a base station (BS) and K users for uplink transmissions. The BS is to receive signals from multiple users over L subcarriers. It is assumed that the BS knows the channel state information (CSI) for signal detection.

The signal vector to be transmitted by user k over L subcarriers is denoted by $\mathbf{s}_k = [s_{k,0} \ \dots \ s_{k,L-1}]^{\text{T}}$, where $s_{k,l} \in \mathcal{S}$ represents the symbol transmitted through the lth subcarrier from user k. Here, $\mathcal{S}$ is the signal constellation. The received signal vector at a BS is given by

$$\mathbf{z} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{s}_k + \mathbf{u}, \quad (1)$$

where $\mathbf{u} \sim \mathcal{CN}(0, N_0\mathbf{I})$ is the background noise vector and $\mathbf{H}_k = \text{diag}(H_{k,0}, \dots, H_{k,L-1})$ is a diagonal channel matrix from user k to the BS with $H_{k,l} = \sum_{p=0}^{P-1} h_{k,p} e^{-j2\pi \frac{pl}{L}}$. Here, $\{h_{k,p}\}$ is the channel impulse response (CIR) from user k to the BS and P is the length of CIR. If the BS has multiple receive antennas, say M antennas, the CIR can be represented by a vector sequence, denoted by $\{\mathbf{h}_{k,p}\}$, where $\mathbf{h}_{k,p} \in \mathbb{C}^M$. In this case, $\mathbf{H}_k$ becomes block diagonal, and its size is $ML \times L$ and the size of $\mathbf{z}$ becomes $ML \times 1$.

A. MC-CDMA

Suppose that each user is to transmit multiple symbols. To avoid inter-user interference (IUI), each user may use a disjoint set of subcarriers to transmit signals. Let $\mathcal{I}_k$ denote the index set of subcarriers, over which user k transmits signals. It is assumed that $\mathcal{I}_k \cap \mathcal{I}_q = \emptyset$ for $k \neq q$. To detect the signals from user k, the subvector of $\mathbf{z}$ taking the elements corresponding to $\mathcal{I}_k$, denoted by $\mathbf{z}_k$, is used. The resulting system becomes an orthogonal frequency division multiple access (OFDMA) system.

In some systems, the allocation of subcarriers to users may not be desirable due to high allocation overhead. Furthermore, if a signal transmitted through a single subcarrier, it may experience severe fading. To avoid these difficulties, it is possible to spread the signal over multiple carriers to exploit a path diversity gain and to eliminate the need of orthogonal subcarrier allocation at the expense of IUI. This results in MC-CDMA [22]. Denote by $\mathbf{C}_k$ the precoding matrix for user k. Then, the received signal vector can be written as

$$\mathbf{z} = \sum_k \mathbf{H}_k \mathbf{C}_k \mathbf{s}_k + \mathbf{u} = \sum_k \mathbf{G}_k \mathbf{s}_k + \mathbf{u}, \quad (2)$$

where $\mathbf{G}_k = \mathbf{H}_k \mathbf{C}_k$. It is noteworthy that in MC-CDMA, the number of symbols per user is not necessary to be L. If a user wants to transmit B symbols over L subcarriers, the size of $\mathbf{s}_k$ becomes $B \times 1$ and that of $\mathbf{C}_k$ becomes $L \times B$. As B decreases, the IUI becomes lower, while the throughput (i.e., the total number of bits over L subcarriers) decreases.

B. Precoded Subcarrier-Index Modulation

Let us consider a single-user system (i.e., $K = 1$) in (1), which results in an OFDM system. For convenience, we assume that user 1 is active. In [13], with sparse $\mathbf{s}_1$, we consider SIM with precoding for OFDM systems (note that SIM *without* precoding for OFDM systems is studied in [8]), which is referred to as precoded SIM in this paper. The main advantage of precoded SIM is that the path diversity can be exploited without using a high-complexity receiver. Note that a low-complexity detector based on the notion of CS will be discussed in Subsection III for $K \geq 1$.

In this subsection, we briefly review SIM in [13]. Consider (1) with $K = 1$ as follows:

$$\mathbf{z} = \mathbf{H}_1 \mathbf{C}_1 \mathbf{s}_1 + \mathbf{u} = \mathbf{G}_1 \mathbf{s}_1 + \mathbf{u},$$

where $\mathbf{C}_1 \in \mathbb{C}^{L \times L}$ and $\mathbf{s}_1$ is Q-sparse. Here, a Q-sparse signal is a vector of Q non-zero elements. Denote by Σ_Q the set of Q-sparse vectors, which is defined as

$$\Sigma_Q = \{\mathbf{x} \,|\, ||\mathbf{x}||_0 = Q\}.$$

Then, $\mathbf{s}_1 \in \Sigma_Q$. In SIM, the message bits are delivered through non-zero data symbols as well as their indices [8]. The number of bits that can be sent through indices is

$$N_I = \lfloor \log_2 \binom{L}{Q} \rfloor,$$

while the number of bits that can be sent by non-zero data symbols is $K \lfloor \log_2 |\mathcal{S}| \rfloor$. For example, if $L = 64$ and $Q = 6$, we have $N_I = 26$ bits. Since each non-zero symbol can represent $\log_2 |\mathcal{S}|$ bits, the total number of bits per user becomes

$$N = \lfloor \log_2 \binom{L}{Q} \rfloor + Q \log_2 |\mathcal{S}|, \quad (3)$$

if $|\mathcal{S}|$ is a power of 2. For convenience, let $\alpha = Q/L$, which is referred to as the sparsity ratio.

In order to see the bit energy of SIM and compare it with that of OFDM, we first consider OFDM, where each element of $\mathbf{s}_1$ is one of $\mathcal{S}$. Denote by E_{sym} the symbol energy. Then, the bit energy becomes $E_{\text{bit}} = E_{\text{sym}} / \log_2 |\mathcal{S}|$. In SIM, since there are Q non-zero elements of $\mathbf{s}_1$, the energy of $\mathbf{s}_1$ becomes $E_{\mathbf{s}} = Q E_{\text{sym}}$.

Since the number of bits of $\mathbf{s}_1$ is N, from (3), the effective bit energy of SIM, denoted by $E_{\rm bit,sim}$, becomes

$$E_{\rm bit,sim} = \frac{E_{\mathbf{s}}}{N} \approx E_{\rm bit} \frac{\log_2 |\mathcal{S}|}{\beta + \log_2 |\mathcal{S}|},$$

where $\beta = L/Q \log_2 \binom{L}{Q}$. For a sufficiently large β (i.e., $\beta \gg \log_2 |\mathcal{S}|$), we have $E_{\rm bit,sim} \approx \log_2 |\mathcal{S}|/\beta \cdot E_{\rm bit}$.

C. SIM for Multi-Carrier Multiple Access

SIM can be used for MC-CDMA. With a small α, the resulting system is called SIMA in this paper. SIMA can be seen as a generalization of precoded SIM to multi-carrier multiple access systems with a few activated subcarriers per user. In SIMA, we assume that $\mathbf{C}_k \in \mathbb{C}^{L \times L}$ and most $s_{k,l}$'s are zero, while information bits can be delivered through non-zero data symbols as well as their indices.

For convenience, let

$$\mathbf{G} = [\mathbf{G}_1 \ \ldots \ \mathbf{G}_K] \text{ and } \mathbf{s} = [\mathbf{s}_1^{\rm T} \ \ldots \ \mathbf{s}_K^{\rm T}]^{\rm T}, \quad (4)$$

where the sizes of $\mathbf{G}$ and $\mathbf{s}$ are $ML \times LK$ and $LK \times 1$, respectively. Furthermore, $\mathbf{s}_k$ is Q-sparse for SIM. Then, from (2), we have

$$\mathbf{z} = \mathbf{G}\mathbf{s} + \mathbf{u}, \quad (5)$$

where $\mathbf{s}$ is KQ-sparse. Furthermore, we consider a real-valued representation. Let

$$\mathbf{r} = \begin{bmatrix} \Re(\mathbf{z}) \\ \Im(\mathbf{z}) \end{bmatrix} \text{ and } \mathbf{n} = \begin{bmatrix} \Re(\mathbf{u}) \\ \Im(\mathbf{u}) \end{bmatrix},$$

where $\mathbf{r}, \mathbf{n} \in \mathbb{R}^{LM}$. In addition, let

$$\mathbf{\Phi}_k = \begin{bmatrix} \Re(\mathbf{G}_k) & -\Im(\mathbf{G}_k) \\ \Im(\mathbf{G}_k) & \Re(\mathbf{G}_k) \end{bmatrix}, \mathbf{x}_k = \begin{bmatrix} \Re(\mathbf{s}_k) \\ \Im(\mathbf{s}_k) \end{bmatrix},$$

$\mathbf{\Phi} = [\mathbf{\Phi}_1 \ \ldots \ \mathbf{\Phi}_K]$, and $\mathbf{x} = [\mathbf{x}_1^{\rm T} \ \ldots \ \mathbf{x}_K]^{\rm T}$. Then, the real-valued representation of $\mathbf{z}$ in (2) becomes

$$\mathbf{r} = \mathbf{\Phi}\mathbf{x} + \mathbf{n}. \quad (6)$$

With (6), in this paper, for SIM, we now consider $\mathbf{x}_k$ (not $\mathbf{s}_k$) which is assumed to be a (real-valued) $\bar{Q}$-sparse vector of length $\bar{L} = 2L$. In this case, the number of bits to be transmitted by SIM per user becomes

$$N_I = \lfloor \log_2 \binom{\bar{L}}{\bar{Q}} \rfloor, \quad (7)$$

which becomes about twice larger than that with complex-valued sparse $\mathbf{s}_k$. Note that $\bar{Q}$ is not necessarily an even[1] number. For convenience, we denote by $\mathcal{X}$ the constellation of the non-zero elements of $\mathbf{x}_k$.

Let us consider an example with $\bar{Q} = 1$ and $\mathcal{X} = \{-\sqrt{E_{\rm bit}}, \sqrt{E_{\rm bit}}\}$ (i.e., binary phase shift keying (BPSK) for non-zero elements). Then, we have

$$N_I = \log_2(\bar{L}) = \log_2 L + 1;$$
$$N = \log_2(\bar{L}) + 1 = \log_2 L + 2.$$

Furthermore, it can be shown that $E_{\rm bit,sim} = E_{\mathbf{s}}/N = E_{\rm bit}/(\log_2 L + 2)$.

[1]For example, if $\mathbf{s}_k = [j \ 0]^{\rm T}$, we have $\mathbf{x}_k = [0 \ 1 \ 0 \ 0]^{\rm T}$, which is 1-sparse, not 2-sparse.

III. COMPRESSIVE SENSING BASED DETECTION

If $\bar{Q} \ll \bar{L}$ or the sparsity ratio, α, is sufficiently small, it is possible to derive a low-complexity detector by exploiting the notion of CS. In this section, we derive a low-complexity detector using the OMP algorithm.

A. Multiuser Detection

The maximum likelihood (ML) detection can be considered to detect $\mathbf{x}$ from $\mathbf{r}$ in (6). Let $\tilde{\mathcal{X}} = \mathcal{X} \cup \{0\}$, which is the extended signal constellation of $\mathcal{X}$ including zero as SIM is employed. Then, the ML detection is given by

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} ||\mathbf{r} - \mathbf{\Phi}\mathbf{x}||^2$$
$$\text{subject to } \mathbf{x} \in \tilde{\mathcal{X}}^{\bar{L}K} \text{ and } \mathbf{x} \in \Sigma_{\bar{Q}K}. \quad (8)$$

Since the length of $\mathbf{x}$ is $\bar{L}K$, the number of signals that have different supports in $\Sigma_{\bar{Q}K}$ is

$$N_S = \binom{K\bar{L}}{K\bar{Q}}.$$

We can easily show that the complexity of the ML detection grows exponentially with K for fixed $\bar{Q}$ and $\bar{L}$. Thus, the ML detection becomes prohibitive for a large K. For example, if $\bar{Q} = 1$, $L = 64$ (or $\bar{L} = 128$), and $K = 10$, we have $N_S = \binom{K\bar{L}}{K} \approx 3 \times 10^{24}$.

B. CS-based Detection

For practical detection schemes, we can exploit the notion of CS under the assumption that $Q \ll L$. Since $\mathbf{s}$ or $\mathbf{x}$ is sparse, from (6), we can consider to solve the following problem [15], [20]:

$$\tilde{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} ||\mathbf{r} - \mathbf{\Phi}\mathbf{x}||_2^2 + \lambda||\mathbf{x}||_1, \quad (9)$$

where $\lambda > 0$ is a design parameter, which can be considered as the Lagrange multiplier. As λ increases, the sparsity constraint is more emphasized. The sparse signal estimation using (9) depends on the RIP condition, which is given by

$$(1 - \epsilon)||\mathbf{s}||^2 \leq ||\mathbf{G}\mathbf{s}||^2 \leq (1 + \epsilon)||\mathbf{s}||^2 \quad (10)$$

or

$$(1 - \epsilon)||\mathbf{x}||^2 \leq ||\mathbf{\Phi}\mathbf{x}||^2 \leq (1 + \epsilon)||\mathbf{x}||^2, \quad (11)$$

where $\epsilon \in (0, 1)$ is the isometry constant. We consider the RIP in Section IV in detail.

The problem in (9) is a convex optimization problem that has a number of algorithms to solve it (e.g., [17]). Unfortunately, its complexity becomes prohibitively high if LM is large. Alternatively, we can consider low-complexity greedy algorithms, e.g., the OMP algorithm [21] to estimate $K\bar{Q}$-sparse signals. In [16], it is shown that the OMP algorithm performs well to estimate sparse signals under certain conditions including the RIP condition.

For convenience, we summarize the conventional OMP algorithm as follows:

**)* Input: $\mathbf{r}$, $\mathbf{\Phi}$, and $\bar{K} = K\bar{Q}$

0) Initialize: $\mathbf{y}^{(0)} = \mathbf{r}$, $\mathbf{x}^{(0)} = \mathbf{0}$, and $\mathcal{T}^{(0)} = \emptyset$

1) for $i = 1 : \bar{K}$
2) $\mathbf{g}^{(i)} = \mathbf{\Phi}^{\mathrm{T}} \mathbf{y}^{(i-1)}$
3) $p^* = \mathrm{argmax}_p \, |[\mathbf{g}^{(i)}]_p| / ||[\mathbf{\Phi}]_p||$
4) $\mathcal{T}^{(i)} = \mathcal{T}^{(i-1)} \cup p^*$
5) $\mathbf{x}^{(i)}_{\mathcal{T}} = [\mathbf{\Phi}_{\mathcal{T}^{(i)}}]^\dagger \mathbf{r}$
6) $\mathbf{y}^{(i)} = \mathbf{r} - \mathbf{\Phi} \mathbf{x}^{(i)}$
7) end;
8) Output: $\mathbf{y}^{(i)}$, $\hat{\mathbf{x}}^{(i)}$

While the OMP algorithm works for any $\bar{K}$-sparse signals, it does not take into account a special property of $\mathbf{s}$ or $\mathbf{x}$. Since $\mathbf{x}$ is a concatenation of K $\bar{Q}$-sparse signal blocks (i.e., the $\mathbf{x}_k$'s), each block is $\bar{Q}$-sparse. Thus, line 3 of the OMP algorithm above needs to be modified to take into account this constraint. The resulting OMP algorithm will be referred to as the constrained OMP (C-OMP) algorithm for convenience.

Denote by $\mathcal{T}_k^{(i)}$ the index set of the selected subcarriers for user k at the ith iteration. We assume that $\mathcal{T}_k^{(0)} = \emptyset$. It is clear that $|\mathcal{T}_k^{(i)}|$ cannot be greater than $\bar{Q}$ as each $\mathbf{x}_k$ is $\bar{Q}$-sparse. Let $\mathcal{I}^{(i)}$ denote the column index of $\mathbf{\Phi}$ at the ith iteration with $\mathcal{I}^{(0)} = \{1, \ldots, \bar{L}K\}$. Then, for the C-OMP algorithm, line 3 is modified as

3a) $p^* = \mathrm{argmax}_{p \in \mathcal{I}^{(i)}} \, |[\mathbf{g}^{(i)}]_p| / ||\mathbf{\Phi}_p||$
3b) $k^* = \lceil \frac{p^*}{\bar{L}} \rceil$
3c) $\mathcal{T}_{k^*}^{(i)} = \mathcal{T}_{k^*}^{(i-1)} \cup p^*$
3d) $\mathcal{T}^{(i)} = \cup_k \mathcal{T}_k^{(i)}$
3e) If $|\mathcal{T}_{k^*}^{(i)}| = \bar{Q}$, $\mathcal{I}^{(i)} = \mathcal{I}^{(i-1)} \setminus \{\bar{L}(k^*-1)+1, \ldots, \bar{L}k^*\}$.

Once the indices of non-zero elements of $\mathbf{s}$ are found by the C-OMP algorithm, the modulated signal signals can be further decided. Let $\mathcal{I}$ denote the (estimated) index set of non-zero elements. In addition, let

$$\bar{\mathbf{r}} = [\mathbf{r}]_{\mathcal{I}}, \bar{\mathbf{x}} = [\mathbf{x}]_{\mathcal{I}}, \text{ and } \bar{\mathbf{\Phi}} = [\mathbf{\Phi}]_{\mathcal{I}}. \tag{12}$$

Then, the values of the non-zero symbols in $\mathbf{s}$ can be detected as

$$\hat{\bar{\mathbf{x}}} = \underset{\bar{\mathbf{x}} \in \mathcal{X}^{2K}}{\mathrm{argmin}} \, ||\bar{\mathbf{r}} - \bar{\mathbf{\Phi}} \bar{\mathbf{x}}||^2. \tag{13}$$

It is also possible to employ the minimum mean squared error (MMSE) detector at the expense of performance.

It is noteworthy that if the index set $\mathcal{I}$ is not correctly estimated, the joint detection with $\mathcal{I}$ cannot provide reasonable decisions. Due to this error propagation problem, the detection in (13) is effective only when the sparse signal estimation is successful. In other words, the performance of the signal detection mainly depends on the sparse signal estimation by the C-OMP algorithm.

IV. PROPERTIES OF SIM FOR MULTI-CARRIER MULTIPLE ACCESS

In this section, we focus on the RIP condition for complex-valued random matrix $\mathbf{G}$. It is noteworthy that most results on RIP conditions are for real-valued random matrices [19], [20, Chap. 5], [24], [25].

We consider the following assumptions.

A1) The elements of $\mathbf{C}_k$ are independent identically distributed (iid) and each element is one of $\{(\pm 1 \pm j)/\sqrt{2L}\}$ with equal probability.

A2) The block diagonal elements of $\mathbf{H}_k$ are iid and each element is a CSCG random variable with mean zero and variance $1/M$.

In general, the frequency-domain channel coefficients can be correlated in OFDM. However, we consider **A2)** for tractable analysis in this section.

For convenience, we first assume that $M = 1$ (i.e., a single receive antenna). Denote by $\mathbf{d}_{k,l}^{\mathrm{H}}$ is the lth row vector of $\mathbf{C}_k$ and define

$$X_l = \sum_{k=1}^{K} H_{k,l} \mathbf{d}_{k,l}^{\mathrm{H}} \mathbf{s}_k = \mathbf{v}_l^{\mathrm{H}} \mathbf{s}, \tag{14}$$

where $\mathbf{v}_l^{\mathrm{H}} = [H_{1,l} \mathbf{d}_{1,l}^{\mathrm{H}} \ \ldots \ H_{K,l} \mathbf{d}_{K,l}^{\mathrm{H}}]$. For given $\mathbf{s}$, under **A1)** and **A2)**, the mean and variance of X_l are

$$\mathbb{E}[X_l] = 0 \text{ and } \mathrm{Var}(X_l) = \frac{||\mathbf{s}||^2}{L}. \tag{15}$$

We consider the Gaussian approximation as

$$X_l \sim \mathcal{CN}\left(0, \frac{||\mathbf{s}||^2}{L}\right). \tag{16}$$

Then, $|X_l|^2 = ||\mathbf{s}||^2/(2L) \cdot \chi_2^2$, where χ_n^2 denotes a chi-squared random variable with n degrees of freedom, and $\sum_l |X_l|^2 = ||\mathbf{s}||^2/(2L) \cdot \chi_{2L}^2$.

We now consider multiple receive antennas, i.e., $M \geq 1$. Under **A2)**, since there are ML X_l's, we have

$$\sum_l |X_l|^2 = \frac{||\mathbf{s}||^2}{2ML} \chi_{2ML}^2. \tag{17}$$

From [26], we have the following inequalities for chi-squared random variables:

$$\begin{aligned} &\Pr(\chi_n^2 - n \geq 2\sqrt{nt} + 2t) \leq e^{-t}; \\ &\Pr(\chi_n^2 - n \leq -2\sqrt{nt}) \leq e^{-t}. \end{aligned} \tag{18}$$

For the upper tail, we have

$$\Pr\left(\sum_l |X_l|^2 - ||\mathbf{s}||^2 \geq \frac{\sqrt{2MLt} + t}{ML} ||\mathbf{s}||^2\right) \leq e^{-t}. \tag{19}$$

Letting $\epsilon = (\sqrt{2MLt} + t)/(ML)$, it follows

$$\Pr\left(\sum_l |X_l|^2 - ||\mathbf{s}||^2 \geq \epsilon ||\mathbf{s}||^2\right) \leq e^{-\frac{ML}{2}\left(\sqrt{1+\frac{\epsilon}{2}} - 1\right)^2}. \tag{20}$$

Since $\sqrt{1+x} \geq 1 + x/2 - x^2/8$, the right-hand-side (RHS) can be bounded as

$$e^{-\frac{ML}{2}\left(\sqrt{1+\frac{\epsilon}{2}} - 1\right)^2} \leq \exp\left(-\frac{ML}{2}\left(\frac{\epsilon}{4} - \frac{\epsilon^2}{32}\right)^2\right).$$

For the lower tail, from (18), we have

$$\Pr\left(\sum_l |X_l|^2 - ||\mathbf{s}||^2 \leq -\sqrt{\frac{2t}{ML}} ||\mathbf{s}||^2\right) \leq e^{-t}. \tag{21}$$

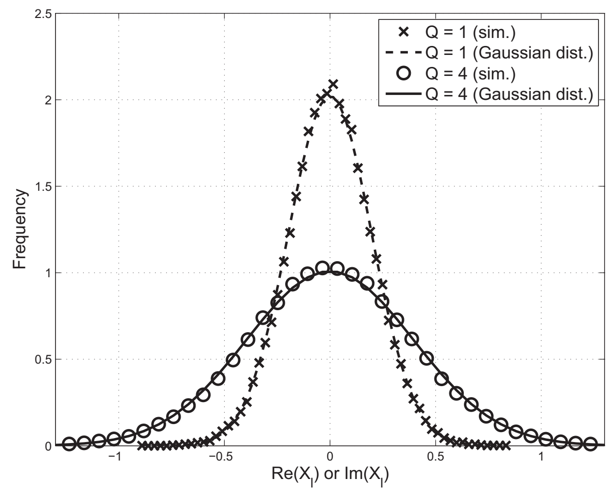

Fig. 1. Histogram of $\Re(X_l)$ and $\Im(X_l)$ when $M = 4$, $L = 64$, $K = 10$.

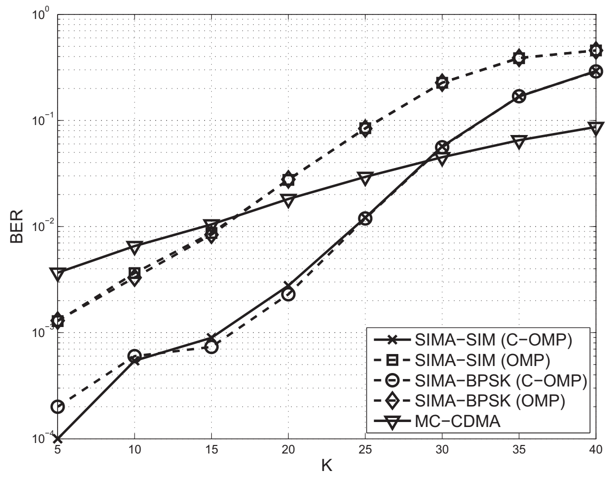

Fig. 2. BER performances of SIMA and MC-CDMA for different numbers of users ($L = 64$, $M = 2$, $P = 3$, and $\mathsf{SNR} = E_{\rm bit}/N_0 = 8$ dB).

Letting $\epsilon = \sqrt{2t/(ML)}$, we have

$$\Pr\left(\sum_l |X_l|^2 - ||\mathbf{s}||^2 \le -\epsilon||\mathbf{s}||^2\right) \le \exp\left(-\frac{ML}{2}\epsilon^2\right). \tag{22}$$

If $\mathbf{u} = 0$, we can see that

$$\sum_l |X_l|^2 = ||\mathbf{z}||^2 = ||\mathbf{Gs}||^2. \tag{23}$$

Thus, from (20) and (22), we can see that $\mathbf{G}$ can satisfy the RIP with an overwhelming probability. In addition, a large ML is desirable for the RIP condition as the bounds in (20) and (22) decrease exponentially with ML with a fixed $\epsilon > 0$.

In Fig. 1, we show histograms of $\Re(X_l)$ and $\Im(X_l)$ from simulations under **A1)** and **A2)** when $M = 4$, $L = 64$, $K = 10$. We can see that $\Re(X_l)$ and $\Im(X_l)$ can be well approximated by Gaussian random variables. Thus, the RIP condition would be satisfied with a high probability for large L or/and M as in (20) and (22).

V. SIMULATION RESULTS

In this section, we present simulation results of SIMA. For convenience, we only consider the case of $\bar{Q} = 1$ (i.e., only one subcarrier is activated for each user) and $\mathcal{X} = \{\pm\sqrt{E_{\rm bit}}\}$ (i.e., BPSK). The SNR in this section is defined as

$$\mathsf{SNR} = \frac{E_{\rm bit}}{N_0}.$$

In order to detect multiple user signals in SIMA, the C-OMP and OMP algorithms are considered. For comparison purposes, we consider MC-CDMA with QPSK signaling. In order to detect multiple users' signals in MC-CDMA, we use the minimum mean squared error (MMSE) detector [27], which is a low-complexity linear detector.

For simulations, we assume that each element of $\mathbf{C}_k$ is independent and uniformly generated from $\{(\pm 1 \pm j)/\sqrt{2L}\}$ (according to **A1)**). In addition, we assume that the pth tap coefficient of the CIR from user k to of the mth antenna at the BS is given by

$$h_{k,p,m} \sim \mathcal{CN}\left(0, \frac{1}{PM}\right),$$

and all the tap coefficients are independent. Note that since the channel gain is normalized, the received signal power is invariant with respect to M, while the dimension of the received signal increases with M.

Fig. 2 shows the BER performances of SIMA and MC-CDMA for different numbers of users when $L = 64$, $M = 2$, $P = 3$, and $\mathsf{SNR} = E_{\rm bit}/N_0 = 8$ dB. In SIMA, there are two different BERs: one for SIM and the other for BPSK. It is shown that the BER of BPSK signals is almost the same as that of SIM signals due to the error propagation. Thus, we will only show the BER performances of SIM signals later. The C-OMP algorithm outperforms the OMP algorithm for all numbers of K. It is also shown that the performances of SIMA are better than that of MC-CDMA if K is sufficiently small.

In Fig. 3, we show BER performances of SIMA and MC-CDMA for various values of SNR when $K = 20$ with $L = 64$, $M = 2$, and $P = 3$. While it can also be confirmed that the C-OMP detector outperforms the OMP-detector, it is observed that SIMA performs better than MC-CDMA at moderate SNRs (≥ 4 dB).

In order to see the impact of P (i.e., the length of CIR) on the performance, simulations are carried out with various values of P and results are shown in Fig. 4 with $L = 64$, $M = 2$, $\mathsf{SNR} = E_{\rm bit}/N_0$ = 8 dB, and $K = 10$. The BER of SIMA decreases with P, which shows that the path diversity gain is exploited by precoding in SIMA.

The CS-based detection can provide good performances if there are more measurements, i.e., M is large. Fig. 5 shows the impact of M on BER performances. It is shown that both MC-CDMA and SIMA have better performances for larger M. It is interesting to see that the performance improvement of SIMA by increasing M is more significant than that of MC-CDMA when the SNR is high. In particular, the gap between the BERs of SIMA and MC-CDMA becomes larger for a larger M and higher SNR.

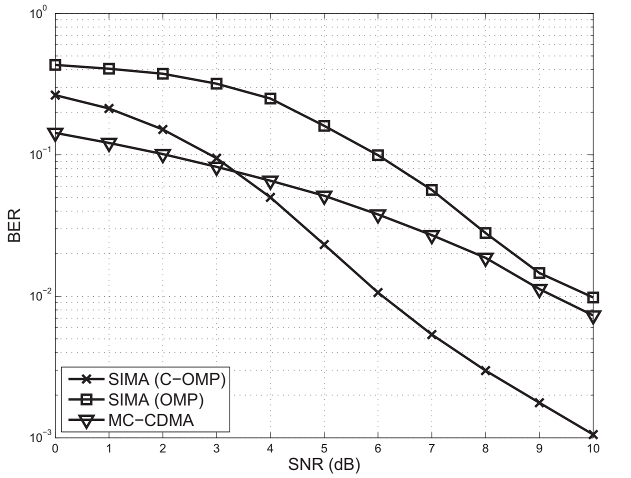

Fig. 3. BER performances of SIMA and MC-CDMA for various values of $\mathsf{SNR} = E_{\rm bit}/N_0$ ($L = 64$, $M = 2$, $P = 3$, and $K = 20$).

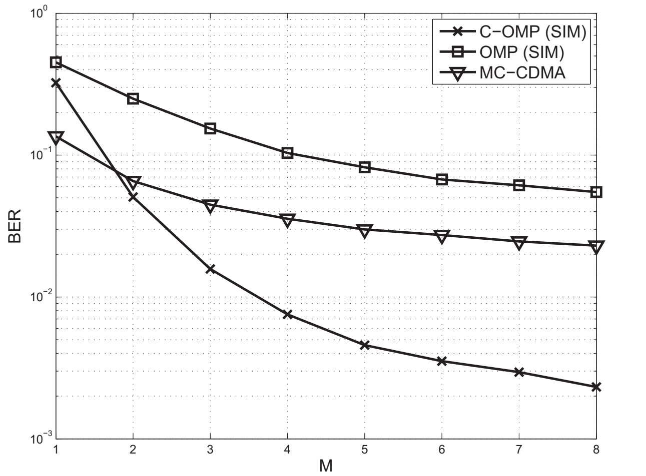

Fig. 5. BER performances of SIMA and MC-CDMA for various values of M ($L = 64$, $P = 4$, $E_{\rm bit}/N_0 = 4$ dB, and $K = 20$).

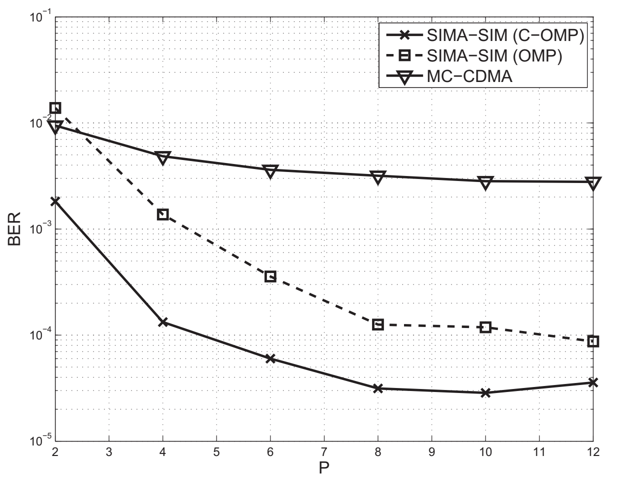

Fig. 4. BER performances of SIMA and MC-CDMA for various values of $\mathsf{SNR} = E_{\rm bit}/N_0$ ($L = 64$, $M = 2$, $\mathsf{SNR} = E_{\rm bit}/N_0$ = 8 dB, and $K = 10$).

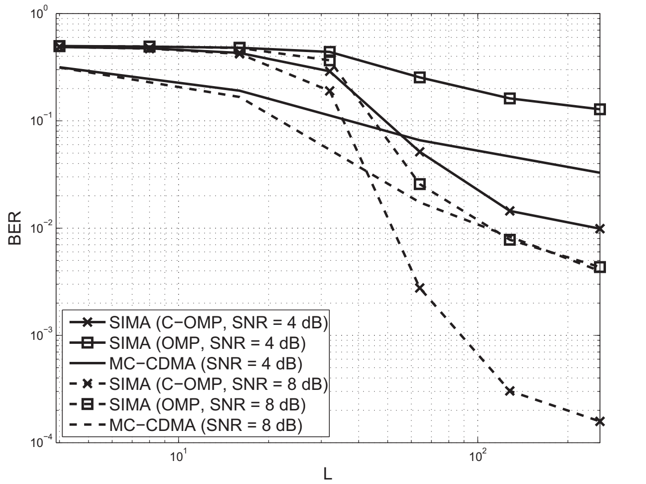

Fig. 6. BER performances of SIMA and MC-CDMA for various values of L ($E_{\rm bit}/N_0 \in \{4, 8\}$ dB, $M = 2$, $P = 3$, and $K = 20$).

Fig. 6 shows the BER performances for different numbers of L with $\mathsf{SNR} = E_{\rm bit}/N_0 = \{4, 8\}$ dB, $M = 2$, $P = 3$, and $K = 20$. It is shown that SIMA can perform much better than MC-CDMA for large L's (e.g., $L = 128$ and 256). While the performance of MC-CDMA is slowly improved by increasing L, that of SIMA is quickly improved. Since L increases, $\mathbf{s}$ becomes more sparse (as $\bar{Q} = 1$ is fixed), which allows the CS-based detection to perform well and result in good performances. Thus, it is desirable to employ SIMA when L is sufficiently large.

In the above results, we consider the same SNR when SIMA and MC-CDMA are compared, which might be fair in terms of the energy efficiency, but not in terms of the spectral efficiency. The number of bits transmitted per user in SIMA with $\bar{Q} = 1$ is $2L + 1$. On the other hand, in MC-CDMA, the number of bits transmitted per user is 2 as QPSK is used. This shows that SIMA can transmit more bits than MC-CDMA. In particular, the spectral efficiency gap between SIMA and MC-CDMA increases with L. In Fig. 7, we show the BER performance in terms of the number of bits per user with various values of L in SIMA and compare it with that of CDMA when $L = 256$. Clearly, with a large L, we can see that SIMA is better than MC-CDMA in terms of energy efficiency (i.e., BER performance for the same SNR) as well as spectral efficiency.

VI. CONCLUSIONS

In this paper, we proposed an application of SIM to multi-carrier multiple access, which results in SIMA. The main advantages of SIMA over other multiple access schemes are energy efficiency (for large systems) as well as low-complexity CS-based detection. We also extended SIM for real-valued representations, which can result in an increase of the throughput by a factor of 2. We modified an existing CS algorithm (i.e., OMP algorithm) for the signal detection in SIMA to take into account a constraint in SIMA. It was shown that the modified CS algorithm for signal detection (i.e., the C-OMP detector) can perform better than the conventional one (i.e., the OMP detec-

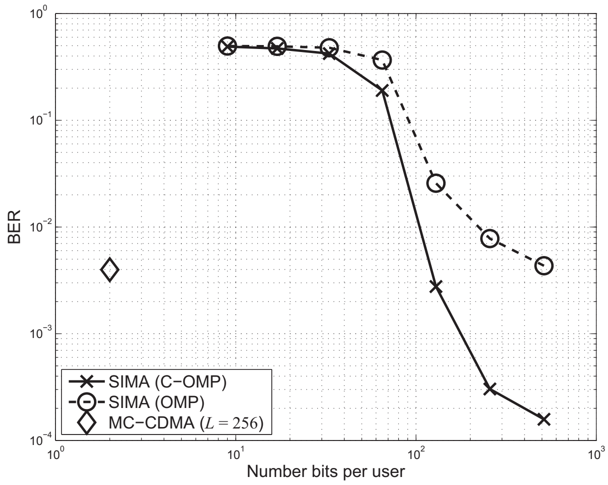

Fig. 7. BER performances of SIMA and MC-CDMA versus the number of bits per user ($E_{\rm bit}/N_0 = 8$ dB, $M = 2$, $P = 3$, and $K = 20$).

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