On the Throughput Comparison between Multi-Channel ALOHA and Compressive Random Access

Jinho Choi
School of Electrical Engineering and Computer Science, GIST
e-mail: jchoi0114@gist.ac.kr

***Abstract*—In this paper, we study the throughput of compressive random access for machine type communications (MTC) and compare it with that of multi-channel ALOHA. In compressive random access, the sparsity of active devices is exploited to use compressive sensing (CS) based low-complexity multiuser detection (MUD) when non-orthogonal spreading codes are used to support a number of devices. For tractable analysis, we consider a high signal-to-noise ratio (SNR) assumption with a collision model as well as flat fading environments. Based on the analysis, we can show that compressive random access is promising (and performs better than multi-channel ALOHA) to support a number of devices with a sufficiently low activity, which might be suitable for MTC.**

***Index Terms*—random access; compressive sensing; multichannel**

I. Introduction

In order to support a number of devices for the Internet of Things (IoT), there has been a growing interest in machine type communications (MTC). MTC is also an important issue in 5th generation (5G) systems [1]. The applications of MTC are diverse from health care to smart grid, where a huge number of devices exist in the system, but only a few of them are active at a particular timing instance. Therefore, random access is suitable for MTC to accommodate a number of devices with a low probability of activity [1]–[3].

In the long term evolution-advanced (LTE-A) system, a random access scheme, called the random access (RACH) procedure, has been proposed for MTC [4]. The RACH procedure is a contention-based random access method, which is similar to the slotted ALOHA protocol [5]. In the RACH procedure, there are multiple preambles in a preamble pool, and a device can select a preamble randomly from the pool and transmit the selected one for access. Finally, it can be connected if there is no collision, i.e., the same preamble is not transmitted by any other devices. Since there are multiple preambles, the RACH procedure can be seen as multi-channel ALOHA [6].

Recently, the notion of compressive sensing (CS) [7]–[9] is employed to exploit the sparsity of active devices for multiuser detection (MUD) in random access [10]–[14]. Among many devices in a system for MTC, only a few of them attempt to access the network by transmitting their preambles. The sparse activity of devices, which can be modeled as a sparse vector, allows the principle of CS to be effectively applied to MUD with low complexity. Among existing compressive random access[1] schemes [11]–[14], there can be two different approaches. In one group of approaches, each device has a unique signature sequence that is used to distinguish its signal from other signals from different devices [11], [12], [14]. The main drawback of this group of approaches is that the number of devices is limited by the number of preambles. Since the performance of CS-based MUD can be worse as more preambles are used, the number of devices in this group of approach would be limited, which is not desirable for MTC in 5G where it is expected to support a number of devices (e.g., 30,000 devices in a system [15]).

In the other group of approaches for compressive random access, an active device can choose a preamble in a set of predetermined preambles (which is common to all devices) and transmit it to connect to an access point (AP) [13], [16]. While the approaches in the second group can support any number of devices, they suffer from collision (as in the RACH procedure) when multiple active devices choose the same preamble.

In this paper, we aim at understanding the throughput of compressive random access and comparing it with that of multi-channel ALOHA[2] in order to see when compressive random access is suitable for MTC For tractable analysis, we assume flat-fading channels and a collision model throughout the paper. From the analysis and simulation results, we can confirm that compressive random access is promising and compressive random access schemes that use a common pool of preambles might be more suitable for MTC to support a number of devices with sparse activity.

Notation: The superscripts T and H denote the transpose and complex conjugate, respectively. The p-norm of a vector $\mathbf{a}$ is denoted by $||\mathbf{a}||_p$ (If $p = 2$, the norm is denoted by $||\mathbf{a}||$ without the subscript). $\mathbb{E}[\cdot]$ and $\mathrm{Var}(\cdot)$ denote the statistical

[1]This term is used in [13] to refer to a random access scheme that allows a receiver to employ low-complexity CS algorithms for MUD.

[2]To the best of our knowledge, there is no work yet where compressive random access is compared with other well-known random access schemes in terms of throughput.

This research was supported by Korea Electric Power Corporation through Korea Electrical Engineering & Science Research Institute (grant number: R15XA03-60) and the GIST Research Institute (GRI) in 2016.

expectation and variance, respectively. $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ represents the distribution of circularly symmetric complex Gaussian (CSCG) random vectors with mean vector $\mathbf{a}$ and covariance matrix $\mathbf{R}$.

II. Multi-Channel ALOHA

Suppose that there are K devices and one AP in a system (e.g., a single cell system). We assume that a given channel is divided into N *orthogonal* channels for multi-channel ALOHA to support connections for K devices of sparse activity. To implement this, we may consider code division multiple access (CDMA). For simplicity, we assume that the bandwidth is sufficiently narrow so that the signals transmitted from active devices experience flat fading. Let M denote the number of active device at a time and assume that each active device can randomly choose one of N orthogonal spreading codes to transmit signals to the AP as a preamble. The set of N orthogonal spreading codes of length N is given by $\mathcal{C} = \{\mathbf{c}_1, \ldots, \mathbf{c}_N\}$, where $|\mathbf{c}_n^{\mathrm{H}} \mathbf{c}_l| = \delta_{n,l}$. Here, $\delta_{n,l}$ denotes the Kronecker delta. Denote by h_k and s_k the channel coefficient and data symbol from device k, respectively. Then, the received signal at the AP is given by

$$\mathbf{y} = \sum_{k \in \mathcal{A}} \mathbf{c}_{l(k)} h_k s_k + \mathbf{n}, \tag{1}$$

where $\mathcal{A}$ represents the index set of active devices, $l(k)$ denotes the index of the code that is chosen by device k, and $\mathbf{n} \sim \mathcal{CN}(0, N_0 \mathbf{I})$ is the background noise. Clearly, $|\mathcal{A}| = M$.

Thanks to the orthogonal codes in $\mathcal{C}$, the AP can recover signals without using MUD. The AP can use a set of N correlators to detect multiple signals from active devices. At the AP, the lth correlator output is given by

$$\begin{aligned} z_l = \mathbf{c}_l^{\mathrm{H}} \mathbf{y} &= \sum_{k \in \mathcal{A}} \mathbf{c}_l^{\mathrm{H}} \mathbf{c}_{l(k)} h_k s_k + \mathbf{c}_l^{\mathrm{H}} \mathbf{n} \\ &= \sum_{k \in \mathcal{A}(l)} h_k s_k + n_l, \end{aligned} \tag{2}$$

where $\mathcal{A}(l)$ is the index set of active devices that choose code l and $n_l = \mathbf{c}_l^{\mathrm{H}} \mathbf{n} \sim \mathcal{CN}(0, N_0)$. The AP can decode the signal from active devices from z_l if there is only one active device in $\mathcal{A}(l)$. On the other hand, if there are more than one active devices in $\mathcal{A}(l)$, collision happens and we may assume that the AP is unable to decode any signals. It is possible to recover the strongest signal if the other signals involved in collision are sufficiently weak thanks to capture effect. However, we ignore this possibility for the sake of simplicity in the performance analysis later.

Note that in multi-channel ALOHA, we may use non-orthogonal spreading codes to have more channels. In this case, the AP has to employ MUD rather than a bank of correlators to detect signals. In this case, the resulting scheme becomes compressive random access provided that CS-based MUD is employed, which is presented in Subsection III-B.

III. Compressive Random Access

In compressive random access, the sparsity of active devices is to be exploited for MUD in random access as mentioned earlier. Thus, in general, we assume a sufficiently low access probability for a large number of devices in compressive random access. In this section, we briefly explain two different compressive random access schemes.

A. Compressive Random Access with Unique Signatures for Devices

For compressive random access, in [12], [14], each device has a unique signature[3] or spreading code. Since the number of devices is large, spreading codes are not necessarily orthogonal to each other. Denote by $\boldsymbol{\phi}_k$ the spreading code for device k. Then, similar to (1), the received signal at the AP becomes

$$\mathbf{y} = \sum_{k \in \mathcal{A}} \boldsymbol{\phi}_k h_k s_k + \mathbf{n}. \tag{3}$$

Let $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1 \ \ldots \ \boldsymbol{\phi}_K]$. In addition, define $a_k = h_k s_k$ for $k \in \mathcal{A}$ and $a_k = 0$ for $k \notin \mathcal{A}$. Then, we have

$$\mathbf{y} = \boldsymbol{\Phi} \mathbf{a} + \mathbf{n}, \tag{4}$$

where $\mathbf{a} = [a_1 \ \ldots \ a_K]^{\mathrm{T}}$ is M-sparse. In the context of CS, $\boldsymbol{\Phi}$ is referred to as the measurement matrix [9], [17].

In compressive random access, the sparsity of $\mathbf{a}$ is exploited to perform MUD for all the signals from M active devices with a low-complexity CS algorithm such as the orthogonal matching pursuit (OMP) algorithm [18].

B. Compressive Random Access with a Common Pool of Signatures

The performance of CS algorithms for MUD depends on the properties of the measurement matrix, $\boldsymbol{\Phi}$. In general, as the number of columns of $\boldsymbol{\Phi}$ increases, the performance of CS algorithms is degraded. In compressive random access, this implies that the performance of MUD becomes poor when K is large. In order to avoid this problem, we may use a common pool of spreading codes as in [16] where a common pool of (non-orthogonal) spreading codes is considered. Each active device can choose one of spreading codes for random access as in multi-channel ALOHA. However, it is not necessary to use orthogonal spreading codes if CS-based MUD is to be employed.

Let $\boldsymbol{\Psi} = [\boldsymbol{\psi}_1 \ \ldots \ \boldsymbol{\psi}_L]$, where L is the number of spreading codes. In general, $L > N$ and the $\boldsymbol{\psi}_l$'s are not mutually orthogonal. An active device can choose one of the spreading codes in $\boldsymbol{\Psi}$ and the received signal at the AP becomes

$$\mathbf{y} = \boldsymbol{\Psi} \mathbf{b} + \mathbf{n}, \tag{5}$$

where $\mathbf{b} = [b_1 \ \ldots \ b_L]^{\mathrm{T}}$. Here, $b_l = \sum_{k \in \mathcal{B}(l)} h_k s_k$, where $\mathcal{B}(l)$ denotes the index set of active devices that use the lth spreading code, $\boldsymbol{\psi}_l$.

This compressive random access scheme also suffers from collision as in multi-channel ALOHA, because the same

[3]Throughout the paper, we assume that signature and spreading code are interchangeable.

spreading code can be chosen by multiple active devices. On the other hand, since this compressive random access scheme can have more spreading codes than that in multi-channel ALOHA (as spreading codes do not need to be mutually orthogonal in compressive random access), it can effectively reduce the probability of collision.

For convenience, the compressive random access scheme where each device has a unique signature (in Subsection III-A) is referred to as CRA-1, while that with a common pool of signature (in this subsection) is referred to as CRA-2. Note that CRA-2 can support much more devices than CRA-1 at the cost of collisions.

IV. Throughput Analysis

In this section, we derive the throughputs of multi-channel ALOHA, CRA-1, and CRA-2 schemes. For tractable analysis, we consider a high signal-to-noise ratio (SNR) assumption. In addition, we assume a collision model for both multi-channel ALOHA and CRA-2.

A. Throughput of Multi-Channel ALOHA

When there are M active devices, the conditional probability that a certain active device can successfully transmit its signal to the AP under the collision model is given by $P_{\rm s}(M) = \left(1-\frac{1}{N}\right)^{M-1}$. Thus, we can have the throughput conditioned on M active devices as follows:

$$T_{\rm A}(M) = M\left(1-\frac{1}{N}\right)^{M-1}. \tag{6}$$

If we assume that a device becomes active independently with an equal access probability, denoted by $p_{\rm a}$, the probability that there are M active devices is given by

$$P_{\rm a}(M) = \binom{K}{M} p_{\rm a}^M (1-p_{\rm a})^{K-M}. \tag{7}$$

From this, the average throughput becomes

$$\begin{aligned} T_{\rm A} &= \mathbb{E}[T_{\rm A}(M)] \\ &= Kp_{\rm a}\left(1-\frac{p_{\rm a}}{N}\right)^{K-1} \approx Kp_{\rm a}e^{-\frac{p_{\rm a}(K-1)}{N}}. \end{aligned} \tag{8}$$

From (8), we can readily see that the maximum throughput of multi-channel ALOHA is M-time higher than that of single-channel ALOHA, which is $1/e$ [5], i.e., $T_{\rm A} \le \frac{N}{e}$.

B. Throughput of Compressive Random Access

The performances of various CS algorithms are generally decided by the properties of $\mathbf{\Phi}$. For example, the coherence of $\mathbf{\Phi}$, which is the maximum of the absolute values of the inner products of two distinct columns of $\mathbf{\Phi}$, i.e.,

$$\mu(\mathbf{\Phi}) = \max_{l\neq m} |\boldsymbol{\phi}_l^{\rm H}\boldsymbol{\phi}_m|, \tag{9}$$

is a performance metric. Note that the coherence is easy to find compared to the restricted isometric constant (RIC) [7], [20]. From [21], for a recovery guarantee (in the absence of noise), a sufficient condition with the coherence is found as

$$M < \tau(\mathbf{\Phi}) = \frac{1}{2}\left(1+\frac{1}{\mu(\mathbf{\Phi})}\right). \tag{10}$$

Therefore, a lower bound on the throughput of CRA-1, where there is no collision, conditioned on M active devices in compressive random access can be obtained as

$$T_{\rm C1}(M) \ge M\mathbb{1}(M < \tau(\mathbf{\Phi})), \tag{11}$$

where $\mathbb{1}(\cdot)$ is the indicator function. In (11), we assume that a CS algorithm cannot recover M signals if M is greater than a certain threshold as in (10) and this event is referred to as the sparsity-outage. In fact, a CS algorithm can recover all the signals or some of the signals although M is greater than $\tau(\mathbf{\Phi})$ (note that the condition in (10) is a sufficient condition). Thus, the conditional throughput in (11) is a lower-bound.

From (7) and (11), a lower-bound on the average throughout of CRA-1 can be found as

$$\begin{aligned} T_{\rm C1} &= \mathbb{E}[T_{\rm C1}(M)] \\ &\ge \sum_{m=1}^{K} m\mathbb{1}(m<\tau(\mathbf{\Phi}))P_{\rm a}(m) \\ &= \sum_{m=1}^{U} m\binom{K}{m}p_{\rm a}^m(1-p_{\rm a})^{K-m} \\ &= \tilde{T}_{\rm C1} \triangleq p_{\rm a}K\sum_{m=0}^{U-1}\binom{K-1}{m}p_{\rm a}^m(1-p_{\rm a})^{K-1-m} \\ &= p_{\rm a}KF(U-1;K-1,p_{\rm a}), \end{aligned} \tag{12}$$

where $U = \lfloor \tau(\mathbf{\Phi}) \rfloor$ and $F(k;n,p) = \sum_{i=0}^{k}\binom{n}{i}p^i(1-p)^{n-i}$ is the cdf of the binomial random variable with parameters n and p. From (12), we can see that U has to be sufficiently larger than $Kp_{\rm a}$ for a reasonably high throughput.

For CRA-2, we need to take into account collisions as well as sparsity-outage. In this case, the conditional throughput for given M active devices becomes

$$T_{\rm C2}(M) \ge M\left(1-\frac{1}{L}\right)^{M-1}\mathbb{1}(M<\tau(\mathbf{\Phi})), \tag{13}$$

As in (12), we can find a lower-bound on the average throughput of CRA-2 as follows:

$$\begin{aligned} T_{\rm C2} &= \mathbb{E}[T_{\rm C2}(M)] \\ &\ge \sum_{m=1}^{K} m\left(1-\frac{1}{L}\right)^{m-1}\mathbb{1}(m<\tau(\mathbf{\Phi}))P_{\rm a}(m) \\ &= Kp_{\rm a}\sum_{m=0}^{U-1}\binom{K-1}{m}\left(p_{\rm a}\left(1-\frac{1}{L}\right)\right)^m(1-p_{\rm a})^{K-1-m} \\ &\ge \tilde{T}_{\rm C2} \triangleq \tilde{T}_{\rm C1}\left(1-\frac{1}{L}\right)^{U-1}. \end{aligned} \tag{14}$$

Comparing (14) with (12), we can see that CRA-2 may have a lower throughput than CRA-1. However, since each device does not need to have a unique spreading code in CRA-2, it can support any number of devices.

According to (11), it is desirable to have a large $\tau(\mathbf{\Phi})$. For a set of spreading sequences, Zadoff-Chu (ZC) sequences can be considered as in LTE-A system [4]. In this case, for a prime N, there are $N-1$ ZC root sequences [22]. For each ZC root sequence, we can have N sequences by circular shifting. These sequences are orthogonal to each other, while the correlation between root sequences is $\frac{1}{\sqrt{N}}$. Thus, the coherence of $\mathbf{\Phi}$ consisting of ZC sequences is $\frac{1}{\sqrt{N}}$. Note that since there are $N(N-1)$ ZC sequences, they can support up to $K = N(N-1)$ devices in CRA-1 or build a pool of $N(N-1)$ signature codes for CRA-2. We can also consider Alltop sequences [23], [24] with prime N. The coherence is also $\frac{1}{\sqrt{N}}$. Since there are N^2 possible Alltop sequences, up to $K = N^2$ devices can be supported in CRA-1 and a pool of up to N^2 signature codes can be used in CRA-2.

Property 1. *Suppose that ZC or Alltop sequences are used for CRA-1 and CRA-2. In addition, let $L = Nc$, where $c \geq 1$ is constant in CRA-2. Then, as $N \to \infty$, $\tilde{T}_{\mathrm{C2}} \to \tilde{T}_{\mathrm{C1}}$.*

Proof: From (14) and (12), we have

$$\frac{\tilde{T}_{\mathrm{C2}}}{\tilde{T}_{\mathrm{C1}}} = \left(1 - \frac{1}{L}\right)^{U-1} \geq 1 - \frac{U-1}{L}, \qquad (15)$$

where the inequality is due to Bernoulli's inequality. For both ZC and Alltop sequences, since $\tau(\mathbf{\Phi}) = \frac{1}{2}\left(1 + \sqrt{N}\right)$,

$$\lim_{N\to\infty} \frac{U-1}{L} = \lim_{N\to\infty} \frac{1+\sqrt{N}}{2Nc} = 0.$$

This shows that $\tilde{T}_{\mathrm{C2}} \geq \tilde{T}_{\mathrm{C1}}$ when $N \to \infty$. On the other hand, from (14) and (12), we always have $\tilde{T}_{\mathrm{C2}} \leq \tilde{T}_{\mathrm{C1}}$. Consequently, as $N \to \infty$, $\tilde{T}_{\mathrm{C2}}$ has to be equal to $\tilde{T}_{\mathrm{C1}}$. ■

According to Property 1, we may prefer CRA-2 to CRA-1 as CRA-2 could support much more devices with the same asymptotic throughput.

Note that both the throughputs of CRA-1 and CRA-2 depend on the properties of $\mathbf{\Phi}$. In general, CRA-1 and CRA-2 suffer from sparsity-outage, while multi-channel ALOHA suffers from collisions as (8). If U is sufficiently large, we can see that the throughputs of CRA-1 and CRA-2 can be higher than that of multi-channel ALOHA.

V. Simulation Results

In this section, we present simulation results when Alltop sequences are used for CRA-1 and CRA-2. For signal recovery, we use the OMP algorithm [18]. Furthermore, we assume that $A = |h_k s_k|$ is fixed for all active devices. The SNR is defined as $\mathrm{SNR} = \frac{A^2}{N_0}$. In CRA-1, we assume that the maximum number of devices is $2N$ (although it can be up to N^2) to avoid a high computational complexity for CS-based MUD. In addition, for the throughput of multi-channel ALOHA, we use (8) which is based on a high SNR assumption.

Fig. 1 shows the throughputs of multi-channel ALOHA, CRA-1, and CRA-2 when $N = 139$, $L = 2N$, $K = 30,000$, and SNR = 20 dB for various values of p_{a}. When p_{a} is sufficiently low, all the random access schemes provide similar performances. However, as p_{a} increases, the performance of multi-channel ALOHA is degraded due to collision, while CRA-1 and CRA-2 perform well until $p_{\mathrm{a}} = 2 \times 10^{-3}$. According to the coherence, we expect to have a guaranteed recovery to detect up to $\lfloor \frac{1+\sqrt{N}}{2} \rfloor = 6$ active devices. However, in practice, we can see that the OMP algorithm performs better than this guarantee and can recover well up to 60 active devices. Then, when $p_{\mathrm{a}} \geq 2 \times 10^{-3}$, both CRA-1 and CRA-2 suffer from sparsity-outage and their throughputs are lower than Kp_{a}. If p_{a} is too high ($> 10^{-2}$), then there are too many active devices (on average, there are more than 300 devices) and the OMP algorithm cannot perform well to recover sparse signals.

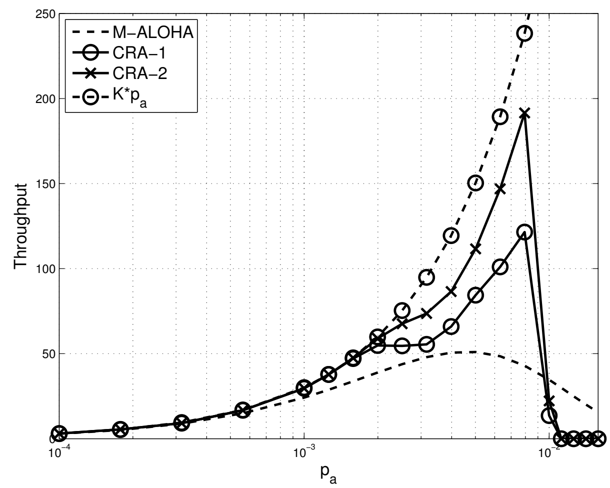

Fig. 1. Throughputs of multi-channel ALOHA (M-ALOHA), CRA-1, and CRA-2 when $N = 139$, $L = 2N$, $K = 30,000$, and SNR = 20 dB for various values of p_{a}.

Note that the maximum throughput of multi-channel ALOHA is $N/e \approx 51.13$, which is also clearly shown in Fig. 1. On the other hand, the throughputs of CRA-1 and CRA-2 can be easily two-time higher than N/e. It is also noteworthy that CRA-2 performs better than CRA-1 in Fig. 1, which is different from the prediction with the lower bounds in (12) and (14). This behavior results from the fact that CRA-2 can have a lower sparsity than CRA-1 on average due to collisions. Thus, there can be more signals to be recovered in CRA-2 than CRA-1 (despite collisions) as the CS-based MUD can perform better with a lower sparsity.

Fig. 2 shows the throughputs of multi-channel ALOHA, CRA-1, and CRA-2 when $L = 2N$, $K = 1000$, $p_{\mathrm{a}} = 0.01$, and SNR = 20 dB for various values of N. As N increases, in multi-channel ALOHA, the performance degradation due to collision decreases, which results in the increase of throughput. We also observe the same behavior for CRA-1 and CRA-2, where a better performance is expected due to more measurements (or a larger N) in CS-based MUD.

To see the impact of the SNR on the throughputs of CRA-1 and CRA-2, we consider various values of SNR with $N = 139$, $L = 2N$, $K = 1000$, and $p_{\mathrm{a}} = 0.01$. The simulation results are shown in Fig. 3. We can see that once the SNR is sufficiently high, the throughput becomes saturated (e.g., the throughput remains unchanged although the SNR increases

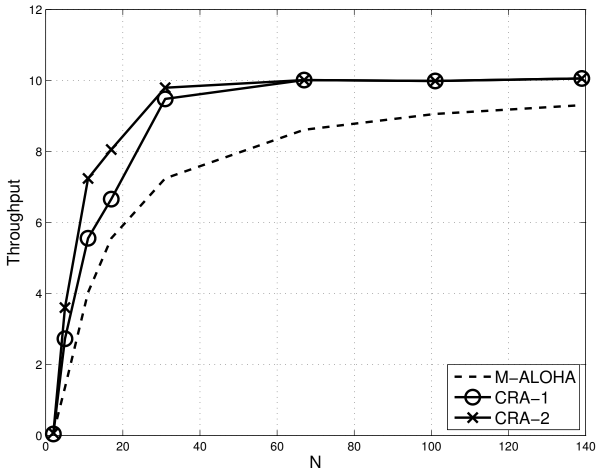

Fig. 2. Throughputs of multi-channel ALOHA (M-ALOHA), CRA-1, and CRA-2 when SNR = 20 dB, $L = 2N$, $K = 1000$, and $p_{\rm a} = 0.01$ for various values of N.

once the SNR is greater than 12 dB). At a high SNR, the CS-based MUD method can successfully recover all the sparse signals as long as the sparsity is small (in this case, there are 10 active devices on average, which is sufficiently small compared to $N = 139$).

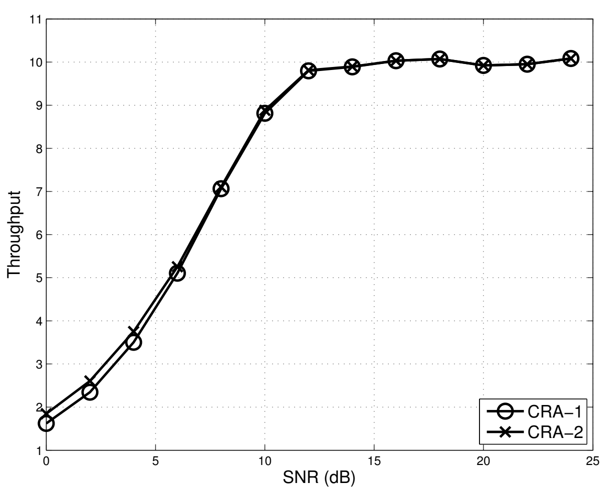

Fig. 3. Throughputs of CRA-1 and CRA-2 when $N = 139$, $L = 2N$, $K = 1000$, and $p_{\rm a} = 0.01$ for various values of SNR.

VI. Conclusions

In this paper, we studied the throughput of compressive random access and compared it with that of multi-channel ALOHA. We showed that compressive random access can perform better than multi-channel ALOHA that uses orthogonal channels thanks to CS-based MUD that can detect multiple signals transmitted through non-orthogonal channels. We also demonstrated that compressive random access with a pool of sprading codes is more suitable for MTC to support a number of devices with a better performance than compressive random access with unique spreading codes for devices.

References

[1] M. Hasan, E. Hossain, and D. Niyato, "Random access for machine-to-machine communication in LTE-advanced networks: issues and approaches," *IEEE Communications Magazine*, vol. 51, pp. 86–93, June 2013.

[2] K.-D. Lee, S. Kim, and B. Yi, "Throughput comparison of random access methods for M2M service over LTE networks," in *GLOBECOM Workshops (GC Wkshps), 2011 IEEE*, pp. 373–377, Dec 2011.

[3] D. Niyato, P. Wang, and D. I. Kim, "Performance modeling and analysis of heterogeneous machine type communications," *IEEE Trans. Wireless Communications*, vol. 13, pp. 2836–2849, May 2014.

[4] 3GPP TR 37.868 V11.0, *Study on RAN improvments for machine-type communications*, October 2011.

[5] B. Bertsekas and R. Gallager, *Data Networks*. Englewood Cliffs: Prentice-Hall, 1987.

[6] Y.-J. Choi, S. Park, and S. Bahk, "Multichannel random access in ofdma wireless networks," *IEEE J. Selected Areas in Communications*, vol. 24, pp. 603–613, March 2006.

[7] E. Candes and T. Tao, "Decoding by linear programming," *IEEE Trans. Inform. Theory*, vol. 51, pp. 4203–4215, Dec 2005.

[8] D. Donoho, "Compressed sensing," *IEEE Trans. Inform. Theory*, vol. 52, pp. 1289–1306, April 2006.

[9] Y. C. Eldar and G. Kutyniok, *Compressed Sensing: Theory and Applications*. Cambridge University Press, 2012.

[10] D. Guo and C.-C. Wang, "Multiuser detection of sparsely spread CDMA," *IEEE J. Selected Areas in Communications*, vol. 26, pp. 421–431, April 2008.

[11] H. Zhu and G. Giannakis, "Exploiting sparse user activity in multiuser detection," *IEEE Trans. Communications*, vol. 59, pp. 454–465, February 2011.

[12] H. F. Schepker, C. Bockelmann, and A. Dekorsy, "Exploiting sparsity in channel and data estimation for sporadic multi-user communication," in *Proc. International Symposium on Wireless Communication Systems (ISWCS 2013)*, pp. 1–5, Aug 2013.

[13] G. Wunder, P. Jung, and C. Wang, "Compressive random access for post-LTE systems," in *Proc. IEEE ICC*, pp. 539–544, June 2014.

[14] G. Wunder, . Stefanovi, P. Popovski, and L. Thiele, "Compressive coded random access for massive MTC traffic in 5G systems," in *2015 49th Asilomar Conference*, pp. 13–17, Nov 2015.

[15] M. Condoluci, M. Dohler, G. Araniti, A. Molinaro, and K. Zheng, "Toward 5G densenets: architectural advances for effective machine-type communications over femtocells," *IEEE Communications Magazine*, vol. 53, pp. 134–141, January 2015.

[16] J. Choi, "On the sparsity for random access in machine type communications under frequency-selective fading," in *Proc. IEEE ICC*, pp. 1–5, May 2016.

[17] E. Candes and M. B. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21–30, 2008.

[18] G. M. Davis, S. G. Mallat, and Z. Zhang, "Adaptive time-frequency decompositions," *Optical Engineering*, vol. 33, no. 7, pp. 2183–2191, 1994.

[19] J. A. Tropp, "Greed is good: algorithmic results for sparse approximation," *IEEE Trans. on Inform. Theory*, vol. 50, pp. 2231–2242, Oct 2004.

[20] A. M. Bruckstein, D. L. Donoho, and M. Elad, "From sparse solutions of systems of equations to sparse modeling of signals and images," *SIAM Review*, vol. 51, no. 1, pp. 34–81, 2009.

[21] D. L. Donoho, M. Elad, and V. N. Temlyakov, "Stable recovery of sparse overcomplete representations in the presence of noise," *IEEE Trans. on Inform. Theory*, vol. 52, pp. 6–18, Jan 2006.

[22] D. Chu, "Polyphase codes with good periodic correlation properties (corresp.)," *IEEE Trans. on Inform. Theory*, vol. 18, pp. 531–532, Jul 1972.

[23] M. Herman and T. Strohmer, "High-resolution radar via compressed sensing," *IEEE Trans. Signal Processing*, vol. 57, pp. 2275–2284, June 2009.

[24] S. Foucart and H. Rauhut, *A Mathematical Introduction to Compressive Sensing*. Springer, 2013.